

Higgsing M2 to D2 with gravity: $\mathcal{N} = 6$ chiral supergravity from topologically gauged ABJM theory

Xiaoyong Chu^{a,*}, Horatiu Nastase^{b,†}, Bengt E.W. Nilsson^{c,‡} and Constantinos Papageorgakis^{d,§}

^a *Service de Physique Théorique, CP225, Université Libre de Bruxelles, Boulevard du Triomphe (Campus de la Plaine), 1050 Bruxelles, Belgium*

^b *Instituto de Física Teórica, UNESP-Universidade Estadual Paulista, R. Dr. Bento T. Ferraz 271, Bl. II, Sao Paulo 01140-070, SP, Brazil*

^c *Fundamental Physics, Chalmers University of Technology, SE-412 96 Göteborg, Sweden*

^d *Department of Mathematics, King's College London, The Strand, London WC2R 2LS, London, UK*

ABSTRACT: We present the higgsing of three-dimensional $\mathcal{N} = 6$ superconformal ABJM type theories coupled to conformal supergravity, so called topologically gauged ABJM theory, thus providing a gravitational extension of previous work on the relation between N M2 and N D2-branes. The resulting $\mathcal{N} = 6$ supergravity theory appears at a chiral point similar to that of three-dimensional chiral gravity introduced recently by Li, Song and Strominger, but with the opposite sign for the Ricci scalar term in the lagrangian. We identify the supersymmetry in the broken phase as a particular linear combination of the supersymmetry and special conformal supersymmetry in the original topologically gauged ABJM theory. We also discuss the higgsing procedure in detail paying special attention to the role played by the U(1) factors in the original ABJM model and the U(1) introduced in the topological gauging.

*E-mail address: xiaoyong.chu@ulb.ac.be

†E-mail address: nastase@ift.unesp.br

‡E-mail address: tfebn@chalmers.se

§E-mail address: costis.papageorgakis@kcl.ac.uk

Contents

1. Introduction and Overview	2
1.1 Some comments on our results	4
1.2 Explicit results	4
2. From three-algebra to bifundamental formulation	6
2.1 The topologically gauged three-algebra theory	7
2.2 Gauging of $U(1)$'s and turning three-algebras into Lie algebras	9
2.3 The topologically gauged ABJM theory in terms of Lie algebras	11
2.3.1 The lagrangian	11
2.3.2 The transformation rules	12
3. Higgsing and expansion around the VEV solution	14
3.1 M2 to D2 for pure ABJM	15
3.1.1 The $SU(N) \times SU(N)$ case	17
3.1.2 The $U(N) \times U(N)$ case	19
3.2 Higgsing including the gravity sector	21
3.2.1 Background: solution, decoupling limit	22
3.2.2 Higgsed supergravity action and the chiral point	23
3.3 The interaction terms	24
3.3.1 Bosonic terms	24
3.3.2 Fermionic terms	27
3.3.3 The B_μ^A, C_μ terms	27
3.4 Supersymmetry rules	30
3.4.1 The effect of the VEV on the supersymmetry of the original fields	30
3.4.2 Supersymmetry for redefined higgsed fields	32
4. Interpreting the supergravity action	34
4.1 Comparison with the chiral gravity and chiral $\mathcal{N}=1$ supergravity literature	35
4.2 Gravity interaction with matter	37
4.3 Higgsing summary and an alternative procedure	38

4.3.1	The higgsing procedure discussed so far	38
4.3.2	An alternative higgsing procedure	39
5.	Conclusions and Outlook	41
A.	Expansion and trace formulas	43
B.	Subleading terms in v	46
B.1	Bosonic terms	47
B.2	Fermionic terms	48

1. Introduction and Overview

The models of Aharony, Bergman, Jafferis and Maldacena (ABJM) [1], describing the physics of N M2-branes on $\mathbb{C}^4/\mathbb{Z}_k$, has received a lot of attention recently. One important step in unravelling the physics of these models is to analyze their relation to D2-branes through a certain Higgs effect connected to a reduction of the background theory from eleven to ten dimensions. Such a higgsing procedure was introduced by Mukhi and Pappageorgakis [2] in the context of the $\mathcal{N} = 8$ BLG theory [3, 4, 5, 6]. This approach was developed further in a series of consecutive papers, see *e.g.* [7, 8] and references therein, where also $\mathcal{N} = 6$ ABJM-type theories were analyzed. The end result contains in many cases Yang-Mills theories appropriate for an interpretation in terms of D2-branes.

The purpose of this paper is to generalize this higgsing procedure to the topologically gauged ABJM-type theories which were recently constructed by Chu and Nilsson in [9] following an earlier attempt for the BLG case in [10]. These topologically gauged theories involve M2-branes whose global symmetries have been gauged by coupling them to superconformal gravity given by a set of Chern-Simons (CS) terms, one for each of the dreibein, Rarita-Schwinger and R-symmetry gauge fields. As will become evident later, the higgsing leads to theories that have some unusual features if viewed as theories of D2-branes. One particularly interesting feature is that the theory, as noted already in [9], ends up at a chiral point in a sense similar to that of Li, Song and Strominger [11]. We also find that the six supersymmetries of this theory arise as linear combinations of the supersymmetry and special superconformal symmetries of the original topologically gauged ABJM theory. However, a number of questions concerning the interpretation of these theories need to be answered. Some of these issues will be addressed in this paper but further study will be required in order to determine their precise role in string/M-theory.

The coupling of superconformal M2-brane theories to three-dimensional gravity also provides an arena where many other interesting questions can be addressed. Gravitational

theories in three dimensions have been extensively studied in a variety of different forms in the past. Examples are for instance topologically massive gravity (TMG) [12] and ‘chiral gravity’ [11]; for some recent work in these directions and further references, see [13, 14, 15, 16]. In these specific cases many of the key features depend on the presence of both an Einstein-Hilbert and a gravitational Chern-Simons term in the lagrangian, the latter of which is of third order in derivatives. Including also an AdS-type cosmological constant term makes these theories exhibit interesting properties related to their spectra of physical states and their boundary conformal field theories. Interestingly enough, whether the propagating modes or the black holes are physical (have positive energy) depends crucially on the sign of the Einstein-Hilbert term in the lagrangian; for a recent discussion of this issue see [16].

As mentioned above, in this paper our starting point will instead be a very particular class of models, namely the ones that arise from the topological gauging of the ABJM theory [9], which provide gravitationally coupled theories preserving superconformal invariance. These models, which have no propagating gravitational degrees of freedom (see *e.g.* [17]), have some features in common with the more familiar Polyakov action in string theory as already pointed out in [10]. In analogy with that case, coupling worldvolume gravity to M2 theories at the conformal fixed point should be done while preserving the conformal invariance, even though this leads to a dependence on the internal metric that in turn prevents its elimination through its field equations.¹ Of course, within string theory there is no independent internal gravity on the branes,² so it must be possible to either eliminate the metric through its field equations, or for the metric to be related to the spacetime or worldvolume fields already present in the theory.

Our main goal will be to gain a better understanding of these models by performing the higgsing of [2], which turns them into theories for stacks of D2-branes coupled to supergravity in a very special way. The supergravity found here is at a ‘chiral point’ similar to that discussed in the bosonic case by Li, Song and Strominger [11] and in the supersymmetric case by Becker, Bruillard and Downes [20]. Thus, as we will see in detail later, the TMG and, in particular, chiral-point gravity models are closely related to the present work as was first noticed in [9]. The fact that the propagating modes have been argued to disappear in chiral gravity [11, 14] ties well in with the fact that the topologically gauged BLG/ABJM theories have no propagating gravitational modes and provides additional incentive for the investigation of the topologically gauged M2-brane theories carried out in this paper. One should, however, be aware of the controversial aspects of these models connected to logarithmic gravity and log-CFT on the boundary, see *e.g.* [21, 22], that might have implications also for the unhiggsed topologically gauged BLG/ABJM theories.

¹This is also the case for the worldsheet metric in the Polyakov action once a Tseytlin term is added. One may also compare this to the situation for D3 branes discussed in [18].

²Like there is for instance in the GDP model [19].

1.1 Some comments on our results

As we will demonstrate below, when reducing the topologically gauged M2-brane theory to D2-branes through the Higgs mechanism of [2], it is clear that one will end up with a lagrangian that also contains a standard Einstein-Hilbert gravity term: When ABJM is coupled to conformal gravity such a higgsing procedure will turn the conformal coupling term of the Ricci scalar to the ABJM scalars, $R|Z|^2$, into the usual EH kinetic term. Giving a VEV to one of the ABJM scalars breaks conformal invariance and generates dynamical gauge fields, resulting in a super Yang-Mills (SYM) theory with a tunable coupling. In addition, the new terms in the scalar potential arising from the topological gauging also produce a non-trivial AdS cosmological constant term. Thus after higgsing the models considered here are precisely of the kind discussed in the contexts of TMG and chiral gravity alluded to above.

However, because we are starting from a conformal theory and turning an interaction term into the leading gravitational kinetic term, the resulting $\mathcal{N} = 6$ theory will have some unusual properties. Especially intriguing is the fact, as was noticed in [9], that this theory automatically sits at a chiral point in the sense of [11], albeit with an opposite sign for the EH term. That the rather involved new sixth order potential terms that arise in the topological gauging gives exactly the correct cosmological constant for the chiral point theory is somewhat non-trivial.

After the higgsing there are a number of parameters which will play an important role. We will therefore present two slightly different sets of limits with different virtues and problems. The physical interpretation of these two limits should probably be kept apart and will not be discussed much. Furthermore, in order to simplify the presentation, we discuss only one of the two sets in the bulk of the paper. In that limit the Newton constant is identified with the inverse VEV, while also giving rise to a background with a constant curvature which does not scale with the VEV. When the VEV becomes large gravity ends up interacting very weakly with matter. We postpone the introduction of the second limit, together with a comparison of the two, until Section 4.3.

An important open question that could perhaps be addressed in this context is whether it is possible to couple these BLG/ABJM models (currently formulated only in a kind of static gauge) to general eleven dimensional supergravity backgrounds, in a fashion similar to that done for a single M2-brane in [23]. A step in this direction might be to couple BLG/ABJM to an M2 worldvolume metric, which one then could imagine as being the worldvolume restriction of a more general spacetime metric. Exactly how such a picture should be related to the case considered in this paper is however not clear. Attempts to couple M2-brane theories to non-gravitational background fields can be found for instance in [24, 25].

1.2 Explicit results

The main result of this work can be summarized in the following expression for the $\mathcal{N} = 6$

chiral supergravity/matter lagrangian, which we obtain after higgsing the topologically gauged ABJM theory of [9] (but before taking the VEV $v \rightarrow \infty$):

$$L = \frac{1}{\kappa^2} L_{\text{CG}} + \frac{1}{\kappa} L_{\text{Int}}^1 + (L_{\text{SYM}} + L_{\text{Int}}^2) + \mathcal{O}(\kappa) , \quad (1.1)$$

with κ the gravitational coupling and where L_{CG} contains the original conformal supergravity terms:³

$$\begin{aligned} \frac{1}{\kappa^2} L_{\text{CG}} = \frac{1}{\kappa^2} & \left[-e(R - 2\Lambda) + \frac{1}{2\mu} \varepsilon^{\mu\nu\lambda} \text{Tr}_\alpha (\tilde{\omega}_\mu \partial_\nu \tilde{\omega}_\lambda + \frac{2}{3} \tilde{\omega}_\mu \tilde{\omega}_\nu \tilde{\omega}_\lambda) \right. \\ & - \frac{1}{2\mu} \varepsilon^{\mu\nu\rho} C_\mu \partial_\nu C_\rho + 8|B_{\mu 4}^{A'}|^2 - 8(qC_\mu + B_\mu)^2 \\ & - \frac{2}{\mu} \varepsilon^{\mu\nu\rho} \text{Tr}_{A'} (B_\mu \partial_\nu B_\rho + \frac{2}{3} B_\mu B_\nu B_\rho) - \frac{4}{\mu} \varepsilon^{\mu\nu\rho} B_{\mu 4}^{A'} \tilde{D}_\nu B_{\rho A'}^4 + \frac{2}{\mu} \varepsilon^{\mu\nu\rho} B_{0\mu} \partial_\nu B_{0\rho} \\ & \left. - \frac{ie}{\mu} \tilde{D}_\mu \tilde{\chi}_\nu^{AB} \gamma^{\rho\sigma} \gamma^{\mu\nu} \tilde{D}_\rho \chi_{\sigma AB} + c.c. + 2i\varepsilon^{\mu\nu\rho} \tilde{\chi}_\mu^{AB} \left(\tilde{D}_\nu + \frac{1}{2\ell} \gamma_\nu \right) \chi_{\rho AB} + c.c. \right] \quad (1.2) \end{aligned}$$

In the above ℓ is the AdS_3 radius ($\Lambda = -\frac{1}{\ell^2}$) and μ is a parameter satisfying $\mu\ell = 1$ as required for a theory sitting at the chiral point [11]. The covariant derivatives \tilde{D} are in general with respect to the spin connection and the various gauge fields that appear in the theory, to be given in detail in due course, while

$$\tilde{D}_\nu B_{\rho A'}^4 \equiv \partial_\nu B_{\rho A'}^4 + iB_{0\nu} B_{\rho A'}^4 - \frac{2}{3} B_{\nu A'}^{B'} B_{\rho B'}^4 . \quad (1.3)$$

Note that the signs in front of the mass terms for the two vector fields in the second line of (1.2) is irrelevant for stability since they are linear in mass. We want to emphasize that in the one of the two limiting procedures where the above expression is valid, the lagrangian describes gravitational excitations around the AdS_3 background and that the action is valid only for small κ , or low energy. Also note that in the strict limit the matter and gravity sectors completely decouple. On the other hand, in the second way of taking the limits the parameters of the theory are all free and tunable, independent of the limiting value $v \rightarrow \infty$. We discuss these two possibilities in more detail in Section 4.3.

The expression (1.2) for the gravity part of the theory is valid in the case of $\text{SU}(N) \times \text{SU}(N)$ matter while for the perhaps more physically relevant case (with a possible gravity dual) $\text{U}(N) \times \text{U}(N)$ the mass term $(qC + B)^2$ is replaced by the interaction term

$$-\frac{4\sqrt{N\mu}}{g_{YM}} \varepsilon^{\mu\nu\rho} (qC_\mu + iB_{0\mu}) F_{\nu\rho}^{+0} . \quad (1.4)$$

The next set of terms in (1.2) contains matter-gravity interactions and is given by

$$\frac{1}{\kappa} L_{\text{Int}}^1 = \frac{1}{\kappa} \left[-e \left(\frac{1}{2} R + 3\mu^2 \right) \tilde{X}_0^4 - i\sqrt{2}eA(\tilde{\psi}_0^{A'} + i\tilde{\psi}_0^{A'+4}) \gamma^{\mu\nu} \left(\tilde{D}_\mu + \frac{1}{2\ell} \gamma_\mu \right) \chi_{\nu A'4} + c.c. \right] . \quad (1.5)$$

³After the higgsing the original $\text{SU}(4)$ unbroken R-symmetry gauge field $B_{\mu B}^A$, $A = 1, \dots, 4$, decomposes into an $\text{SU}(3)$ part $B_{\mu B'}^{A'}$, $A' = 1, \dots, 3$, a $\mathbf{3}$ of $\text{SU}(3)$ $B_{\mu 4}^{A'}$, and a singlet $B_{\mu 4}^4 = iB_{0\mu}$. The six supersymmetry generators are in the $\mathbf{3} + \bar{\mathbf{3}}$ of the R-symmetry group $\text{SU}(3)$.

We comment on the presence of terms linear in \tilde{X}_0^4 during their derivation in Section 3.3.1.

For the $\mathcal{O}(0)$ terms, L_{SYM} is the usual 2+1d $\mathcal{N} = 8$ SYM action⁴ (minimally coupled to supergravity) with coupling g_{YM} , although the maximal supersymmetry is broken down to $\mathcal{N} = 6$ in the full theory due to the coupling to supergravity. Finally, the second set of matter-gravity interaction terms are given by

$$\begin{aligned} L_{\text{Int}}^2 = & -2\mu g_{YM} e f^{abc} X_a^{A'+4} X_b^{A'} X_c^8 - \frac{3}{2} e \mu^2 \tilde{X}_0^4 \tilde{X}_0^4 - \frac{e}{8} \left(\frac{1}{2} R + 3\mu^2 \right) (\tilde{X}_0^I \tilde{X}_0^I + X_a^I X_a^I) \\ & + \frac{3i\mu}{2} e \left[\bar{\psi}_0^{A'} \tilde{\psi}_0^{A'} + \bar{\psi}_0^{A'+4} \tilde{\psi}_0^{A'+4} \right] + \frac{i\mu}{2} e \left[\bar{\psi}_a^{A'} \psi_a^{A'} + \bar{\psi}_a^{A'+4} \psi_a^{A'+4} \right] \\ & + \frac{i\mu}{2} e \left[5(\bar{\psi}_0^4 \tilde{\psi}_0^4 + \bar{\psi}_0^8 \tilde{\psi}_0^8) - 3(\bar{\psi}_a^4 \psi_a^4 + \bar{\psi}_a^8 \psi_a^8) \right], \end{aligned} \quad (1.6)$$

where \tilde{X}_0^4 , X_a^8 , $X^{A'}$ and $X^{A'+4}$ are real fields coming from the four complex bifundamental fields Z^A in the ABJM model. The various coefficients in the lagrangian reproduced here depend on which of the two sets of limits is used and if the parameters are taken to their limiting values or not.

The rest of this paper is organized as follows: We begin in Section 2 by rewriting the topologically gauged $\mathcal{N} = 6$ model of [9] in terms of bifundamental fields in order to make contact with the original formulation of the ABJM theories. We present the details of the Higgs mechanism in Section 3, where we first review the gauge theory results and include a discussion on how the mechanism affects the abelian factors, which seems to have been omitted in the literature. We then separately address the gravity and interaction sectors, as well as the supersymmetry transformations. In Section 4 we discuss the resulting $\mathcal{N} = 6$ supergravity theory coupled to SYM and compare it with the pure chiral gravity of [11] as well as its $\mathcal{N} = 1$ relative discussed in [20].

For readers who are interested in the main points of the higgsing, the process is summarized in Section 4.3 where we sketch the relevant steps while skipping most of the technical details. We also postpone the discussion and detailed comparison of the two limits to the end, while using only one of them in the bulk of the paper to increase readability. Some concluding comments are collected in Section 5. A number of useful identities and calculational results can be found in Appendices A and B.

2. From three-algebra to bifundamental formulation

The lagrangian and transformation rules of the topologically gauged ABJM theory constitute the starting point for the discussion in the remaining sections. Therefore, we begin in Section 2.1 with a brief account of this theory as constructed in [9]. This is followed in Section 2.2 by a detailed description of the steps needed for turning the results of [9] into a form more suitable for the higgsing which is the main goal of this paper. The lagrangian and transformation rules we find after performing these steps are presented in Section 2.3.

⁴The eight supersymmetries occur here for the same reasons as in the higgsed ABJM theory without gravity [1].

We would also like to establish some nomenclature for the different theories appearing throughout this paper. The theory of [9] called the “topologically gauged ABJM theory” actually refers to a wide class of models, namely all those constructed from a complex three-algebra according to the classification of [26, 27]. Here we will restrict this class to those without $U(1)$ factors and as a result our starting point will contain the $SU(N) \times SU(N)$ but not the $U(N) \times U(N)$ ABJM models. The latter is usually obtained from the former by performing an extra step where some $U(1)$ factors are gauged, as already described in [1, 28].⁵ In our case however this will be done after the topological gauging (which is gravitational) and will in fact be carried out in an entirely different way compared to ABJM. We will be precise about which of the two cases we are analyzing at each point.

2.1 The topologically gauged three-algebra theory

The formulation of the ungauged ABJM type models presented in [1] makes use of ordinary gauge fields and Lie algebras. However, in the original work of Bagger-Lambert [3, 4, 5] and Gustavsson [6] the construction of the $\mathcal{N} = 8$ (BLG) theory is in terms of three-algebras. Subsequently, an alternative formulation of the $\mathcal{N} = 6$ ABJM type theories in terms of such three-algebras was given in [30, 31]. In [10, 9] the topologically gauged M2-brane models were all constructed in this three-algebra language while the higgsing, to be performed later in this paper, is more conveniently implemented in the ordinary gauge field and Lie algebra formulation. We will therefore make the effort of rewriting the topologically gauged ABJM type theories of [9] in this more conventional language before subjecting them to the higgsing procedure.

The formulation of ABJM theory obtained in [30] (see also [31]) in terms of four-indexed structure constants is based on complex three-algebra generators T^a and their triple product. For gauge theories with $\mathcal{N} = 6$ supersymmetry the associated structure constants and the trace are defined by

$$[T^a, T^b; T_c] = f^{ab}{}_{cd} T^d, \quad \text{Tr}(T^a T_b) = \delta_b^a, \quad (2.1)$$

where the structure constants are antisymmetric in both the upper and lower pair of indices. All indices are moved up or down by complex conjugation. For further details on the three-algebra and the lagrangian reproduced below we refer the reader to [9].

The topologically gauged M2-brane theories [10, 9] are gravitationally coupled theories preserving the superconformal symmetry. All global symmetries of the M2 matter theories are gauged in the process. Those preserving $\mathcal{N} = 6$ supersymmetry are given in terms of the following lagrangian [9]:

$$\begin{aligned} L = & L_{sugra}^{conf} + L_{ABJM}^{cov} + \frac{1}{2} \epsilon^{\mu\nu\rho} C_\mu \partial_\nu C_\rho \\ & + i A e \bar{\chi}_\mu^{BA} \gamma^\nu \gamma^\mu \Psi_{Aa} (\tilde{D}_\nu \bar{Z}_B^a - \frac{i}{2} A \bar{\chi}_{\nu BC} \Psi^{Ca}) + c.c. \\ & + i \epsilon^{\mu\nu\rho} (\bar{\chi}_\mu^{AC} \chi_{\nu BC}) Z_a^B \tilde{D}_\rho \bar{Z}_A^a + c.c. \end{aligned}$$

⁵For a recent discussion on $U(1)$ factors see [29].

$$\begin{aligned}
& -iA(\bar{f}^{\mu AB}\gamma_\mu\Psi_{Aa}\bar{Z}_B^a + \bar{f}_{AB}^\mu\gamma_\mu\Psi^{Aa}Z_a^B) \\
& -\frac{\epsilon}{8}\tilde{R}|Z|^2 \\
& +\frac{i}{2}|Z|^2\bar{f}_{AB}^\mu\chi_\mu^{AB} \\
& +2ieAf_{cd}^{ab}(\bar{\chi}_{\mu AB}\gamma^\mu\Psi^{d[B})Z_a^{D]}Z_b^AZ_c^{\bar{D}} + c.c. \\
& -i\epsilon^{\mu\nu\rho}(\bar{\chi}_{\mu AB}\gamma_\nu\chi_\rho^{CD})(Z_a^AZ_b^B\bar{Z}_C^c\bar{Z}_D^d)f_{cd}^{ab} \\
& +\frac{i}{4}\epsilon^{\mu\nu\rho}(\bar{\chi}_{\mu AB}\gamma_\nu\chi_\rho^{AB})(Z_a^CZ_b^D\bar{Z}_C^c\bar{Z}_D^d)f_{cd}^{ab} \\
& -\frac{i}{16}\epsilon\epsilon^{ABCD}(\bar{\Psi}_{Aa}\Psi_{Bb})\bar{Z}_C^a\bar{Z}_D^b + c.c. \\
& +\frac{i}{16}e(\bar{\Psi}_{Db}\Psi^{Db})|Z|^2 - \frac{i}{4}e(\bar{\Psi}_{Db}\Psi^{Bb})\bar{Z}_B^aZ_a^D \\
& +\frac{i}{8}e(\bar{\Psi}_{Db}\Psi^{Da})\bar{Z}_B^bZ_a^B + \frac{3i}{8}e(\bar{\Psi}_{Db}\Psi^{Ba})\bar{Z}_B^bZ_a^D \\
& -\frac{i}{16}eA(\bar{\chi}_{\mu AB}\gamma^\mu\Psi^{Bb})|Z|^2Z_a^A - \frac{i}{4}eA(\bar{\chi}_{\mu AB}\gamma^\mu\Psi^{Db})Z_a^AZ_b^B\bar{Z}_D^a + c.c. \\
& -\frac{i}{4}\epsilon^{\mu\nu\rho}(\bar{\chi}_{\nu AB}\gamma_\rho\chi_\mu^{CD})Z_a^AZ_b^B\bar{Z}_C^a\bar{Z}_D^b + \frac{i}{64}\epsilon^{\mu\nu\rho}(\bar{\chi}_{\nu AB}\gamma_\rho\chi_\mu^{AB})|Z|^4 \\
& +\frac{1}{8}ef_{cd}^{ab}|Z|^2Z_a^CZ_b^D\bar{Z}_C^c\bar{Z}_D^d + \frac{1}{2}ef_{cd}^{ab}Z_a^BZ_b^CZ_e^D\bar{Z}_B^e\bar{Z}_C^c\bar{Z}_D^d \\
& +\frac{5}{12\cdot 64}e|Z|^6 - \frac{1}{32}e|Z|^2Z_b^AZ_a^C\bar{Z}_C^b\bar{Z}_A^a + \frac{1}{48}eZ_a^AZ_b^BZ_d^C\bar{Z}_B^d\bar{Z}_A^a\bar{Z}_C^c, \tag{2.2}
\end{aligned}$$

where

$$\begin{aligned}
L_{ABJM}^{cov} = & -e(\tilde{D}_\mu Z_a^A)(\tilde{D}^\mu \bar{Z}_A^a) - \frac{1}{2}(ie\bar{\Psi}^{Aa}\gamma^\mu\tilde{D}_\mu\Psi_{Aa} + ie\bar{\Psi}_{Aa}\gamma^\mu\tilde{D}_\mu\Psi^{Aa}) \\
& -ie f_{cd}^{ab}\bar{\Psi}^{Ad}\Psi_{Aa}Z_b^B\bar{Z}_B^c + 2ie f_{cd}^{ab}\bar{\Psi}^{Ad}\Psi_{Ba}Z_b^B\bar{Z}_A^c \\
& -\frac{i}{2}\epsilon\epsilon^{ABCD}f_{cd}^{ab}\bar{\Psi}^{Ac}\Psi^{Bd}Z_a^CZ_b^D - \frac{i}{2}\epsilon\epsilon^{ABCD}f_{ab}^{cd}\bar{\Psi}_{Ac}\Psi_{Bd}\bar{Z}_C^a\bar{Z}_D^b \\
& -eV + \frac{1}{2}\epsilon^{\mu\nu\lambda}(f_{cd}^{ab}A_\mu^d{}_b\partial_\nu A_\lambda^c{}_a + \frac{2}{3}f_{gc}^{bd}f_{gf}^{ae}A_\mu^a{}_bA_\nu^c{}_dA_\lambda^e{}_f), \tag{2.3}
\end{aligned}$$

$$V = \frac{2}{3}\Upsilon^{CD}{}_{Bd}\tilde{\Upsilon}_{CD}{}^{Bd}, \tag{2.4}$$

$$\Upsilon^{CD}{}_{Bd} = f_{cd}^{ab}Z_a^CZ_b^D\bar{Z}_B^c + f_{cd}^{ab}\delta^{[C}{}_BZ^{D]}{}_aZ^E{}_b\bar{Z}_E^c \tag{2.5}$$

and

$$\begin{aligned}
L_{sugra}^{conf} = & \frac{1}{2}\epsilon^{\mu\nu\rho}\text{Tr}_\alpha(\tilde{\omega}_\mu\partial_\nu\tilde{\omega}_\rho + \frac{2}{3}\tilde{\omega}_\mu\tilde{\omega}_\nu\tilde{\omega}_\rho) - 2\epsilon^{\mu\nu\rho}\text{Tr}_A(B_\mu\partial_\nu B_\rho + \frac{2}{3}B_\mu B_\nu B_\rho) \\
& -ie^{-1}\epsilon^{\alpha\mu\nu}\epsilon^{\beta\rho\sigma}(\tilde{D}_\mu\bar{\chi}_\nu^{AB}\gamma_\beta\gamma_\alpha\tilde{D}_\rho\chi_{\sigma AB}). \tag{2.6}
\end{aligned}$$

Here $B_\mu{}^A{}_B$ is an $\text{SO}(6)$ R-symmetry gauge field, $A = \pm\sqrt{2}$ and $|Z|^2$ stands for

$$Z_a^A\bar{Z}_A^a = \text{Tr}(\bar{Z}_A, Z^A), \tag{2.7}$$

while the action of the \tilde{D} -covariant derivative on a spinor is given by⁶

$$\tilde{D}_\mu\Psi^{Aa} = \partial_\mu\Psi^{Aa} + \frac{1}{4}\tilde{\omega}_{\mu\alpha\beta}\gamma^{\alpha\beta}\Psi^{Aa} + B_{\mu B}^A\Psi^{Ba} + \tilde{A}_{\mu b}^a\Psi^{Ab} + qC_\mu\Psi^{Aa}. \tag{2.8}$$

From the work of [9] we know that $q^2 = \frac{1}{16}$ in order for the theory to possess six local special conformal as well as ordinary supersymmetries.

In the above formulas we have also used the definitions

$$f^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho}\tilde{D}_\nu\chi_\rho, \tag{2.9}$$

$$\tilde{\omega}_{\mu\alpha\beta} = \omega_{\mu\alpha\beta}(e) - \frac{i}{2}(\bar{\chi}_\mu\gamma_\beta\chi_\alpha - \bar{\chi}_\mu\gamma_\alpha\chi_\beta - \bar{\chi}_\alpha\gamma_\mu\chi_\beta), \tag{2.10}$$

⁶Here we follow the three-algebra literature conventions, with the gauge fields being anti-hermitian.

i.e. $\tilde{\omega}_{\mu\alpha\beta} = \tilde{\omega}_{\mu\alpha\beta}(e, \chi)$ is the usual spin connection of supergravity in the second order formalism.

The novel features of this lagrangian are the conformal coupling term between the curvature scalar and the scalar fields $-\frac{\epsilon}{8}R|Z|^2$, as well as the new terms in the sixth order scalar potential (*i.e.* the last two lines in the lagrangian (2.2) above). These terms will play an important role later.

2.2 Gauging of U(1)'s and turning three-algebras into Lie algebras

Having given the three-algebra formulation of the topologically gauged theory, we will now switch to a Lie algebra formulation in order to facilitate the higgsing procedure presented in the next section. We also give a detailed discussion of the various ways U(1) factors appear in the theories analyzed here.

To find the Lie algebra version of these theories we need to replace the triple product and trace of the abstract three-algebra by equivalent expressions in terms of matrices. We will follow the procedure presented in [30]. As an example, consider the following term already analyzed in [30]:

$$-if^{ab}_{cd}\bar{\Psi}^{Ad}\Psi_{Aa}Z_b^B\bar{Z}_B^c \rightarrow -i\text{Tr}(\bar{\Psi}^A, f^{ab}_{cd}\Psi_{Aa}Z_b^B\bar{Z}_B^c T^d) \rightarrow -i\text{Tr}(\bar{\Psi}^A, [\Psi_A, Z^B; \bar{Z}_B]) , \quad (2.11)$$

where fields without three-algebra indices have been contracted by an element from the algebra. We will finally express everything in terms of the following $N \times N$ matrix realization of the three-algebra

$$\text{Tr}(\bar{X}, Y) = \text{tr}(X^\dagger Y) , \quad [X, Y; \bar{Z}] = \lambda(XZ^\dagger Y - YZ^\dagger X) , \quad (2.12)$$

where $\lambda = \frac{2\pi}{k}$. Thus, for example,

$$Z_a^A \bar{Z}_A^a = \text{Tr}(\bar{Z}_A, Z^A) = \text{tr}(Z_A^\dagger Z^A) , \quad (2.13)$$

Applying the above procedure on one of the new interaction terms of the topologically gauged theory we get:

$$\begin{aligned} \frac{1}{8}ef^{ab}_{cd}|Z|^2 Z_a^C Z_b^D \bar{Z}_C^c \bar{Z}_D^d &\rightarrow \frac{1}{8}e|Z|^2 \text{Tr}(\bar{Z}_D, [Z^C, Z^D; \bar{Z}_C]) \\ &\rightarrow \frac{1}{8}\lambda e|Z|^2 \text{tr}(Z_D^\dagger Z^C Z_C^\dagger Z^D - Z_D^\dagger Z^D Z_C^\dagger Z^C) . \end{aligned}$$

In the next subsection we will use the definition of the trace to introduce one more parameter which will then show up in the last form of the interaction term above.

It is important to note that the procedure of [30] only leads to an $\text{SU}(N) \times \text{SU}(N)$ gauge group [28]. Hence one needs to introduce two abelian factors in order to turn $\text{SU}(N) \times \text{SU}(N)$ into $\text{U}(N) \times \text{U}(N)$, while also keeping the U(1) associated with C_μ and preserving $\mathcal{N} = 6$ supersymmetry. This U(1)-extended formulation will require that the terms in the lagrangian related to the C_μ field be accompanied by terms containing two

abelian fields with opposite CS-level k' (independent of the one in the non-abelian sector), denoted \hat{A}^L and \hat{A}^R . The introduction of \hat{A}^L, \hat{A}^R means that at the end these fields can be combined with the nonabelian \tilde{A}^a_b to give the desired $U(N) \times U(N)$ gauge fields leading to the topologically gauged $U(N) \times U(N)$ ABJM model.

Instead of repeating the rather extensive calculations of [9] for this new situation, we will employ a simpler method to introduce additional abelian Chern-Simons fields without destroying any of the six local supersymmetries. We first split C_μ as $C_\mu = E_\mu + \frac{\alpha}{q}P_\mu$ and note that nothing changes in the check of supersymmetry if we define the variations of E_μ, P_μ as $\delta E_\mu = \delta C_\mu$ and $\delta P_\mu = 0$. Replacing C_μ in the covariant derivative gives

$$\tilde{D}_\mu \psi^{Aa} = \partial_\mu \psi^{Aa} + \frac{1}{4}\tilde{\omega}_{\mu\alpha\beta}\gamma^{\alpha\beta}\psi^{Aa} + B_{\mu B}^A \psi^{Ba} + \tilde{A}_{\mu b}^a \psi^{Ab} + qE_\mu \psi^{Aa} + \alpha P_\mu \psi^{Aa}. \quad (2.14)$$

Eventually we also want to replace the CS term for C_μ without affecting its supersymmetry variation. Hence we consider

$$\delta\left(\frac{1}{2}\varepsilon^{\mu\nu\rho}C_\mu\partial_\nu C_\rho\right) = \delta\left(\frac{1}{2}\varepsilon^{\mu\nu\rho}E_\mu\partial_\nu E_\rho\right) + \frac{\alpha}{q}\varepsilon^{\mu\nu\rho}P_\mu\partial_\nu(\delta E_\rho) \quad (2.15)$$

and redefine P_μ by setting $P_\mu = \hat{A}_\mu^L - \hat{A}_\mu^R$. Recalling that $\delta P^\mu = 0$, any $\delta\hat{A}_\mu^L = \delta\hat{A}_\mu^R$ will do since it does not appear explicitly in the supersymmetry variations. The requirement that when we switch to the Lie algebra formulation the $\hat{A}_\mu^L, \hat{A}_\mu^R$ terms appear in the same way in the covariant derivative as the $SU(N)$ pieces fixes $\alpha = 1$. We then replace the CS term for the C_μ field, $\frac{1}{2}\varepsilon^{\mu\nu\rho}C_\mu\partial_\nu C_\rho$, with

$$\frac{1}{2}\varepsilon^{\mu\nu\rho}E_\mu\partial_\nu E_\rho + \frac{k'}{4\pi}\varepsilon^{\mu\nu\rho}\left(\hat{A}_\mu^L\partial_\nu\hat{A}_\rho^L - \hat{A}_\mu^R\partial_\nu\hat{A}_\rho^R\right) = \frac{1}{2}\varepsilon^{\mu\nu\rho}E_\mu\partial_\nu E_\rho + \frac{k'}{4\pi}\varepsilon^{\mu\nu\rho}P_\mu\partial_\nu(\hat{A}_\rho^L + \hat{A}_\rho^R). \quad (2.16)$$

In order to keep the supersymmetry invariance, we equate its variation with (2.15), obtaining

$$\begin{aligned} -i\delta\hat{A}_\mu^L &= -i\delta\hat{A}_\mu^R = \frac{2\pi}{k'}\frac{\alpha}{q}\delta E \\ &= \frac{2\pi}{k'}(\bar{\epsilon}_{AB}\gamma_\mu\Psi^{Aa}Z_a^B - \bar{\epsilon}^{AB}\gamma_\mu\Psi_{Aa}\bar{Z}_B^a) + \frac{4\pi}{k'}(\bar{\epsilon}_g^{AD}\chi_{\mu BD} - \bar{\epsilon}_{gBD}\chi_\mu^{AD})Z_a^B\bar{Z}_A^a. \end{aligned} \quad (2.17)$$

Finally, we rename E_μ as C_μ , and note that in effect, we have introduced the two new fields $\hat{A}_\mu^L, \hat{A}_\mu^R$, on top of the C_μ field.

Thus in the case corresponding to $U(N) \times U(N)$ we will make use of the covariant derivative

$$\tilde{D}_\mu \Psi^{Aa} = \partial_\mu \Psi^{Aa} + \frac{1}{4}\tilde{\omega}_{\mu\alpha\beta}\gamma^{\alpha\beta}\Psi^{Aa} + B_{\mu B}^A \Psi^{Ba} + qC_\mu \Psi^{Aa} + \tilde{A}_{\mu b}^a \Psi^{Ab} + (\hat{A}_\mu^L - \hat{A}_\mu^R)\delta_b^a \Psi^{Ab}, \quad (2.18)$$

and add a related Chern-Simons term to the lagrangian

$$\frac{k'}{4\pi}\varepsilon^{\mu\nu\lambda}(\hat{A}_\mu^L\partial_\nu\hat{A}_\lambda^L - \hat{A}_\mu^R\partial_\nu\hat{A}_\lambda^R)\frac{1}{N}\text{tr}(\mathbb{1}_{N\times N}). \quad (2.19)$$

Note that here we have introduced a factor $1 = \frac{1}{N} \text{tr}(\mathbb{1}_{N \times N})$ that will be useful later when combining the abelian fields with the $SU(N)$ ones to form a $U(N) \times U(N)$ ABJM theory.

A further comment on notation is in order: When we next move to the Lie algebra formulation we will switch from the three-algebra conventions with anti-hermitian generators to the more familiar ones with hermitian generators, in order to connect with the usual ABJM analysis. We will treat the $U(N) \times U(N)$ gauge fields coming from the three-algebra, and the corresponding part of the covariant derivative separately, but for the $SU(4)$ and $U(1)$ fields $B^A_B, C, \hat{A}^L, \hat{A}^R$ this corresponds to multiplying with an extra factor of i .

2.3 The topologically gauged ABJM theory in terms of Lie algebras

The higgsing procedure will involve taking limits in several parameters at the same time. One of these is the CS-level parameter $\lambda = \frac{2\pi}{k}$. However, there is also a natural way to introduce a (super-)gravitational coupling, while retaining the supersymmetry of the lagrangian. We will call this coupling g_M , keeping in mind that it could come from string theory after reduction to 10d. This can be achieved by multiplying all traces by g_M^2 and then rescaling the whole lagrangian by $\frac{1}{g_M^2}$, thus ensuring that the pure ABJM part remains unmodified. Alternatively, one could just insert a g_M^2 on the RHS of the trace identification (2.12) and then rescale the lagrangian by $\frac{1}{g_M^2}$. In this manner one obtains

$$\begin{aligned} \frac{1}{8} e f^{ab} |Z|^2 Z_a^C Z_b^D \bar{Z}_C^c \bar{Z}_D^d &\rightarrow \frac{1}{8} e |Z|^2 \text{Tr}(\bar{Z}_D, [Z^C, Z^D; \bar{Z}_C]) \\ &\rightarrow \frac{1}{8} g_M^2 \lambda e |Z|^2 \text{tr}(Z_D^\dagger Z^C Z_C^\dagger Z^D - Z_D^\dagger Z^D Z_C^\dagger Z^C), \end{aligned}$$

where in the last expression $|Z|^2$ refers to the matrix trace tr .

Note that while L_{ABJM}^{cov} involves a single trace over gauge indices, L_{sugra}^{conf} has no trace and hence the interaction terms are no-trace as well as single-, double- and triple-trace. This will have important implications for the results of the higgsing procedure and in particular for how the various limits affect the terms in the lagrangian.

2.3.1 The lagrangian

Following the procedure described so far, the resulting Lie algebra lagrangian is

$$L = \frac{1}{g_M^2} L_{sugra}^{conf} + L_{ABJM}^{cov} + \frac{1}{2g_M^2} \epsilon^{\mu\nu\rho} C_\mu \partial_\nu C_\rho \quad (2.20)$$

$$+ iA e \bar{\chi}_\mu^{BA} \gamma^\nu \gamma^\mu \text{tr}(\Psi_A (\tilde{D}_\nu Z_B^\dagger - \frac{i}{2} A (\bar{\chi}_{\nu BC} \Psi^C)^\dagger)) + c.c. \quad (2.21)$$

$$\begin{aligned} &+ i\epsilon^{\mu\nu\rho} (\bar{\chi}_\mu^{AC} \chi_{\nu BC}) \text{tr}(Z^B \tilde{D}_\rho Z_A^\dagger) + c.c. \\ &- iA (\bar{f}^{\mu AB} \gamma_\mu \text{tr}(\Psi_A Z_B^\dagger) + \bar{f}_{AB}^\mu \gamma_\mu \text{tr}(\Psi^\dagger{}^A Z^B)) \\ &- \frac{e}{8} \tilde{R} |Z|^2 \end{aligned} \quad (2.22)$$

$$\begin{aligned} &+ \frac{i}{2} |Z|^2 \bar{f}_{AB}^\mu \chi_\mu^{AB} \\ &+ 2i\lambda e A \bar{\chi}_{\mu AB} \gamma^\mu \text{tr}(\Psi^{[B} (Z^{D]} Z_D^\dagger Z^A - Z^{[A} Z_D^\dagger Z^{D]}) + c.c. \end{aligned}$$

$$\begin{aligned}
& -i\lambda\epsilon^{\mu\nu\rho}(\bar{\chi}_{\mu AB}\gamma_\nu\chi_\rho^{CD})\text{tr}(Z_D^\dagger Z^A Z_C^\dagger Z^B - Z_D^\dagger Z^B Z_C^\dagger Z^A) \\
& + \frac{i}{4}\lambda\epsilon^{\mu\nu\rho}(\bar{\chi}_{\mu AB}\gamma_\nu\chi_\rho^{AB})\text{tr}(Z_D^\dagger Z^C Z_C^\dagger Z^D - Z_D^\dagger Z^D Z_C^\dagger Z^C) \\
& - \frac{i}{16}g_M^2 e\epsilon^{ABCD}\text{tr}(\bar{\Psi}_A Z_C^\dagger)\text{tr}(\Psi_B Z_D^\dagger) + c.c.
\end{aligned} \tag{2.23}$$

$$+ \frac{i}{16}g_M^2 e\text{tr}(\bar{\Psi}_D^\dagger \Psi^D)|Z|^2 - \frac{i}{4}g_M^2 e\text{tr}(\bar{\Psi}_D^\dagger \Psi^B)\text{tr}(Z_B^\dagger Z^D) \tag{2.24}$$

$$+ \frac{i}{8}g_M^2 e\text{tr}(\bar{\Psi}_D Z_B^\dagger)\text{tr}(\Psi^{\dagger D} Z^B) + \frac{3i}{8}g_M^2 e\text{tr}(\bar{\Psi}_D Z_B^\dagger)\text{tr}(\Psi^{\dagger B} Z^D) \tag{2.25}$$

$$\begin{aligned}
& - \frac{i}{16}g_M^2 e A \bar{\chi}_{\mu AB} \gamma^\mu \text{tr}(\Psi^{\dagger B} Z^A)|Z|^2 - \frac{i}{4}g_M^2 e A \bar{\chi}_{\mu AB} \gamma^\mu \text{tr}(\Psi^{\dagger D} Z^B)\text{tr}(Z^A Z_D^\dagger) + c.c. \\
& \tag{2.26}
\end{aligned}$$

$$\begin{aligned}
& - \frac{i}{4}g_M^2 \epsilon^{\mu\nu\rho}(\bar{\chi}_{\nu AB}\gamma_\rho\chi_\mu^{CD})\text{tr}(Z^A Z_C^\dagger)\text{tr}(Z^B Z_D^\dagger) + \frac{i}{64}g_M^2 |Z|^4 \epsilon^{\mu\nu\rho}(\bar{\chi}_{\nu AB}\gamma_\rho\chi_\mu^{AB}) \\
& + \frac{1}{8}g_M^2 \lambda e|Z|^2\text{tr}(Z_D^\dagger Z^C Z_C^\dagger Z^D - Z_D^\dagger Z^D Z_C^\dagger Z^C) \\
& + \frac{1}{2}g_M^2 \lambda e\text{tr}(Z^D Z_B^\dagger)\text{tr}(Z_D^\dagger Z^B Z_C^\dagger Z^C - Z_D^\dagger Z^C Z_C^\dagger Z^B) \\
& \tag{2.27}
\end{aligned}$$

$$\begin{aligned}
& + \frac{5}{12 \cdot 64}g_M^4 e|Z|^6 - \frac{1}{32}g_M^4 e|Z|^2\text{tr}(Z^A Z_C^\dagger)\text{tr}(Z^C Z_A^\dagger) \\
& + \frac{1}{48}g_M^4 e\text{tr}(Z^A Z_C^\dagger)\text{tr}(Z^B Z_A^\dagger)\text{tr}(Z^C Z_B^\dagger). \\
& \tag{2.28}
\end{aligned}$$

We do not find it necessary to give the expressions for L_{sugra}^{conf} and L_{ABJM}^{cov} , as the former has no gauge indices and hence stays the same while the latter was already given in this form in [30]. Note that $\lambda = \frac{2\pi}{k}$ multiplied all the three-algebra f_{cd}^{ab} terms and hence also naturally multiplies all f^{abc} terms in the lagrangian above, obtained when performing the traces.

2.3.2 The transformation rules

In the case of the supersymmetry transformations, everything is straightforwardly done in the same fashion, except for the gauge field which will be dealt with separately. The explicit transformations in the three-algebra language are

$$\delta e_\mu^\alpha = i\bar{\epsilon}_{gAB}\gamma^\alpha\chi_\mu^{AB}, \tag{2.29}$$

$$\delta\chi_\mu^{AB} = \tilde{D}_\mu\epsilon_g^{AB}, \tag{2.30}$$

$$\begin{aligned}
\delta B_{\mu B}^A &= \frac{i}{e}(\bar{f}^{\nu AC}\gamma_\mu\gamma_\nu\epsilon_{gBC} - \bar{f}_{BC}^\nu\gamma_\mu\gamma_\nu\epsilon_g^{AC}) \\
&+ \frac{i}{4}(\bar{\epsilon}_{BD}\gamma_\mu\Psi_a^{(D}Z_a^A) - \bar{\epsilon}^{AD}\gamma_\mu\Psi_{a(D}\bar{Z}_B^a) \\
&- \frac{i}{2}(\bar{\epsilon}_g^{AC}\chi_{\mu DC}Z_a^D\bar{Z}_B^a - \bar{\epsilon}_{gBC}\chi_\mu^{DC}Z_a^A\bar{Z}_D^a) \\
&+ \frac{i}{8}\bar{\epsilon}_B^A(\bar{\epsilon}_g^{EC}\chi_{\mu DC} - \bar{\epsilon}_{gDC}\chi_\mu^{EC})Z_a^D\bar{Z}_E^a \\
&+ \frac{i}{8}(\bar{\epsilon}_g^{AD}\chi_{\mu BD} - \bar{\epsilon}_{gBD}\chi_\mu^{AD})|Z|^2, \\
& \tag{2.31}
\end{aligned}$$

$$\delta Z_a^A = i\bar{\epsilon}^{AB}\Psi_{Ba}, \tag{2.32}$$

$$\begin{aligned}
\delta\Psi_{Bd} &= \gamma^\mu\epsilon_{AB}(\tilde{D}_\mu Z_d^A - iA\bar{\chi}_\mu^{AD}\Psi_{Dd}) \\
&+ f_{cd}^{ab}Z_a^C Z_b^D \bar{Z}_B^c \epsilon_{CD} - f_{cd}^{ab}Z_a^A Z_b^C \bar{Z}_C^c \epsilon_{AB}
\end{aligned}$$

$$+\frac{1}{4}Z_c^C Z_d^D \bar{Z}_B^c \epsilon_{CD} + \frac{1}{16}|Z|^2 Z_d^A \epsilon_{AB}, \quad (2.33)$$

$$\begin{aligned} \delta \tilde{A}_\mu^c{}_d &= -i(\bar{\epsilon}_{AB} \gamma_\mu \Psi^{Aa} Z_b^B - \bar{\epsilon}^{AB} \gamma_\mu \Psi_{Ab} \bar{Z}_B^a) f_{ad}^{bc} \\ &\quad - 2i(\bar{\epsilon}_g^{AD} \chi_{\mu BD} - \bar{\epsilon}_{gBD} \chi_\mu^{AD}) Z_b^B \bar{Z}_A^a f_{ad}^{bc}, \end{aligned} \quad (2.34)$$

$$\begin{aligned} \delta \hat{A}_\mu^R &= \delta \hat{A}_\mu^L = \frac{2\pi}{k'} (\bar{\epsilon}_{AB} \gamma_\mu \Psi^{Aa} Z_a^B - \bar{\epsilon}^{AB} \gamma_\mu \Psi_{Aa} \bar{Z}_B^a) \\ &\quad + \frac{4\pi}{k'} (\bar{\epsilon}_g^{AD} \chi_{\mu BD} - \bar{\epsilon}_{gBD} \chi_\mu^{AD}) Z_a^B \bar{Z}_A^a, \end{aligned} \quad (2.35)$$

$$\begin{aligned} \delta C_\mu &= -iq(\bar{\epsilon}_{AB} \gamma_\mu \Psi^{Aa} Z_a^B - \bar{\epsilon}^{AB} \gamma_\mu \Psi_{Aa} \bar{Z}_B^a) \\ &\quad - 2iq(\bar{\epsilon}_g^{AD} \chi_{\mu BD} - \bar{\epsilon}_{gBD} \chi_\mu^{AD}) Z_a^B \bar{Z}_A^a, \end{aligned} \quad (2.36)$$

where $\epsilon_m^{AB} = A \epsilon_g^{AB} = \epsilon^{AB}$.

Leaving the gauge transformation law aside for the moment, by multiplying Z_a^A, Ψ_{Ba} with T^a and writing everything in terms of traces and triple brackets according to (2.12), while also introducing g_M^2 , we get

$$\delta e_\mu^\alpha = i \bar{\epsilon}_{gAB} \gamma^\alpha \chi_\mu^{AB}, \quad (2.37)$$

$$\delta \chi_\mu^{AB} = \tilde{D}_\mu \epsilon_g^{AB}, \quad (2.38)$$

$$\begin{aligned} \delta B_\mu^A{}_B &= \frac{i}{e} (\bar{f}^{\nu AC} \gamma_\mu \gamma_\nu \epsilon_{gBC} - \bar{f}_{BC}^\nu \gamma_\mu \gamma_\nu \epsilon_g^{AC}) \\ &\quad + \frac{i}{4} g_M^2 (\bar{\epsilon}_{BD} \gamma_\mu \text{tr}(\Psi^\dagger(D) Z^A) - \bar{\epsilon}^{AD} \gamma_\mu \text{tr}(\Psi_{(D} Z_B^\dagger)) \\ &\quad - \frac{i}{2} g_M^2 (\bar{\epsilon}_g^{AC} \chi_{\mu DC} \text{tr}(Z^D Z_B^\dagger) - \bar{\epsilon}_{gBC} \chi_\mu^{DC} \text{tr}(Z^A Z_D^\dagger)) \\ &\quad + \frac{i}{8} g_M^2 \delta_B^A (\bar{\epsilon}_g^{EC} \chi_{\mu DC} - \bar{\epsilon}_{gDC} \chi_\mu^{EC}) \text{tr}(Z^D Z_E^\dagger) \\ &\quad + \frac{i}{8} g_M^2 (\bar{\epsilon}_g^{AD} \chi_{\mu BD} - \bar{\epsilon}_{gBD} \chi_\mu^{AD}) |Z|^2, \end{aligned} \quad (2.39)$$

$$\delta Z^A = i \bar{\epsilon}^{AB} \Psi_B, \quad (2.40)$$

$$\begin{aligned} \delta \Psi_B &= \gamma^\mu \epsilon_{AB} (\tilde{D}_\mu Z^A - i A \bar{\chi}_\mu^{AD} \Psi_D) \\ &\quad + \lambda (Z^C Z_B^\dagger Z^D - Z^D Z_B^\dagger Z^C) \epsilon_{CD} - \lambda (Z^A Z_C^\dagger Z^C - Z^C Z_C^\dagger Z^A) \epsilon_{AB} \\ &\quad + \frac{1}{4} g_M^2 \text{tr}(Z^C Z_B^\dagger) Z^D \epsilon_{CD} + \frac{1}{16} g_M^2 |Z|^2 Z^A \epsilon_{AB} \\ \delta C_\mu &= -g_M^2 q (\bar{\epsilon}_{AB} \gamma_\mu \text{tr}(\Psi^\dagger Z^B) - \bar{\epsilon}^{AB} \gamma_\mu \text{tr}(\Psi_A Z_B^\dagger)) \\ &\quad - 2g_M^2 q (\bar{\epsilon}_g^{AD} \chi_{\mu BD} - \bar{\epsilon}_{gBD} \chi_\mu^{AD}) \text{tr}(Z^B Z_A^\dagger). \end{aligned} \quad (2.41)$$

We can now return to the transformation law for the gauge fields. These are obtained by requiring that the transformation law of the covariant derivative remains unchanged under the replacement⁷

$$\begin{aligned} T^a D_\mu Z_a^A &= T^a (\partial_\mu Z_a^A - \tilde{A}_\mu^b{}_a Z_b^A) \Rightarrow \\ D_\mu Z^A &= \partial_\mu Z^A + i \hat{A}_\mu^L Z^A - Z^A i \hat{A}_\mu^R \end{aligned} \quad (2.42)$$

used in the pure ABJM case. This implies that

$$T^d \delta \tilde{A}_\mu^c{}_d F_c^D = -i \delta \hat{A}_\mu^L F^D + i F^D \delta \hat{A}_\mu^R, \quad (2.43)$$

⁷As advertised, we now work with hermitian gauge fields, hence the field C_μ introduced in [9] appears as iC_μ in the covariant derivatives *etc.*

where we used a general object F^D instead of the Z^A in order to facilitate the identification of the transformation rules. The LHS of (2.43) gives

$$\begin{aligned}
& -iT^d(\bar{\epsilon}_{AB}\gamma_\mu\Psi^{Aa}Z_b^B - \bar{\epsilon}^{AB}\gamma_\mu\Psi_{Ab}\bar{Z}_B^a)f^{bc}_{ad}F_c^D \\
& -2i(\bar{\epsilon}_g^{AD}\chi_{\mu BD} - \bar{\epsilon}_{gBD}\chi_\mu^{AD})T^dZ_b^B\bar{Z}_A^af^{bc}_{ad}F_c^D \\
\rightarrow & -i(\bar{\epsilon}_{AB}\gamma_\mu[Z^B, F^D; \Psi^A] - \bar{\epsilon}^{AB}\gamma_\mu[\Psi_A, F^D; \bar{Z}_B]) \\
& -2i(\bar{\epsilon}_g^{AD}\chi_{\mu BD} - \bar{\epsilon}_{gBD}\chi_\mu^{AD})[Z^B, F^D; \bar{Z}_A] \\
\rightarrow & -i\lambda(\bar{\epsilon}_{AB}\gamma_\mu(Z^B\Psi^\dagger{}^AF^D - F^D\Psi^\dagger{}^AZ^B) - \bar{\epsilon}^{AB}\gamma_\mu(\Psi_A Z_B^\dagger F^D - F^D Z_B^\dagger \Psi_A)) \\
& -2i\lambda(\bar{\epsilon}_g^{AD}\chi_{\mu BD} - \bar{\epsilon}_{gBD}\chi_\mu^{AD})(Z^B Z_A^\dagger F^D - F^D Z_A^\dagger Z^B)
\end{aligned} \tag{2.44}$$

and by matching with the RHS of (2.43) we obtain

$$\begin{aligned}
\delta\hat{A}_\mu^L &= -\lambda(\bar{\epsilon}_{AB}\gamma_\mu Z^B\Psi^\dagger{}^A - \bar{\epsilon}^{AB}\gamma_\mu\Psi_A Z_B^\dagger) - 2\lambda(\bar{\epsilon}_g^{AD}\chi_{\mu BD} - \bar{\epsilon}_{gBD}\chi_\mu^{AD})Z^B Z_A^\dagger \\
\delta\hat{A}_\mu^R &= -\lambda(\bar{\epsilon}_{AB}\gamma_\mu\Psi^\dagger{}^A Z^B - \bar{\epsilon}^{AB}\gamma_\mu Z_B^\dagger \Psi_A) - 2\lambda(\bar{\epsilon}_g^{AD}\chi_{\mu BD} - \bar{\epsilon}_{gBD}\chi_\mu^{AD})Z_A^\dagger Z^B.
\end{aligned} \tag{2.45}$$

We can combine this with the abelian factors

$$\begin{aligned}
\delta\hat{A}_\mu^R &= \delta\hat{A}_\mu^L = \frac{2\pi}{k'}\left(\bar{\epsilon}_{AB}\gamma_\mu\text{tr}(\Psi^\dagger{}^A Z^B) - \bar{\epsilon}^{AB}\gamma_\mu\text{tr}(\Psi_A Z_B^\dagger)\right) \\
&+ \frac{4\pi}{k'}(\bar{\epsilon}_g^{AD}\chi_{\mu BD} - \bar{\epsilon}_{gBD}\chi_\mu^{AD})\text{tr}(Z^B Z_A^\dagger),
\end{aligned} \tag{2.46}$$

which, with the definition $k' = kN$, leads to the desired $U(N)$ gauge fields [28]:

$$A_\mu^{L/R} = \hat{A}_\mu^{L/R} - \hat{A}_\mu^{L/R} \mathbb{1}_{N \times N}. \tag{2.47}$$

3. Higgsing and expansion around the VEV solution

Having recast the theory to our preferred form we now wish to investigate its expansion around a particularly simple background, namely one in which a real component of the ABJM scalars Z^A develops a VEV, v . This triggers the Higgs mechanism for CS-matter theories introduced in [2], which turns a parity-invariant pair of CS gauge fields into a single dynamical YM field. In the topologically gauged context there are also chiral CS terms related to the R-symmetry $U(1) \times SU(4)$, which include C_μ . These latter fields will also experience a Higgs effect but of a more conventional type.

During this procedure, following [1, 8, 7] for the pure ABJM case, one would like to keep $v\frac{2\pi}{k} = v\lambda$ fixed, while taking $v \rightarrow \infty$, $\lambda \rightarrow 0$. We will see however that this must be supplemented with a condition on g_M , which will be $g_M v \sqrt{N} \equiv \bar{g}_M = \text{fixed}$ or $g_M = \frac{\bar{g}_M}{v\sqrt{N}} \rightarrow 0$. This is the set of limits that will be used for the rest of this section. However, in Section 4 we will compare them to an alternative set of limits (*i.e.* having a different set of fixed parameters) as mentioned already in the introduction. We work with arbitrary values of N throughout.

For clarity of presentation we split this section into several parts: We begin with a review for the higgsing of the pure ABJM theory, then analyze the gravity part of the action and the new features that arise when the gravitational fields are introduced into the covariant derivatives. Finally we address the interactions and supersymmetry transformation rules.

3.1 M2 to D2 for pure ABJM

The ABJM model, describing N M2-branes on a $\mathbb{C}^4/\mathbb{Z}_k$ singularity, is an $\mathcal{N} = 6$ $U(N) \times U(N)$ CS gauge theory with bifundamental matter. However, as discussed in the previous section, the theory that we here relate to the three-algebra formulation of [30] is its $SU(N) \times SU(N)$ version [26, 28]. One particular example is the $SU(2) \times SU(2)$ model that has $\mathcal{N} = 8$ supersymmetry and is equivalent to the BLG-theory, as shown by Van Raamsdonk [32]. When comparing the situation with and without the two $U(1)$ factors the higgsing leads to a subtlety that has not been fully appreciated in the literature and which we explore in the following.

As we will demonstrate later in this section when the full theory is considered, it is possible to find a set of background solutions to the topologically gauged ABJM models parametrized by a real VEV v . Thus we would like to see what happens to the lagrangian (2.20)-(2.28) under a perturbation schematically of the form

$$Z^A = v\delta^{A4} + z^A, \quad (3.1)$$

with $A = 1, \dots, 4$, or more precisely in terms of the real parts⁸

$$Z^A = v\delta^{A4} \mathbb{1}_{N \times N} + \frac{1}{\sqrt{2}}X^A + i\frac{1}{\sqrt{2}}X^{A+4}. \quad (3.2)$$

This will cause the theory to break both the $SU(4)$ R-symmetry and the quiver gauge symmetry $G \times G$ to the diagonal G , with G either $SU(N)$ or $U(N)$. In addition there is the abelian gauge symmetry associated with the field C_μ which will mix with the other abelian gauge fields during the higgsing process. These issues will be elaborated upon below when the two different choices for G are discussed one at a time.

As already mentioned, this involves taking the limits $v, k \rightarrow \infty$, with $\frac{v}{k}$ fixed. In the spacetime orbifold picture of [33, 1], which is dual to the $U(N) \times U(N)$ ABJM model, setting $Z^{1,2,3} = 0$ reduces to $\mathbb{C}/\mathbb{Z}_{k \rightarrow \infty}$ with⁹

$$Z^4 \rightarrow Z^4 e^{\frac{2\pi i}{k}} \simeq Z^4 \left(1 + 2\pi i \frac{1}{k} + \dots\right) \simeq Z^4 + 2\pi i \frac{Z^4}{k}. \quad (3.3)$$

Expanding around $Z^4 = v + i0$ with $\frac{v}{k}l_{11}^{3/2} \equiv R$ we obtain that

$$Z^4 l_{11}^{3/2} \rightarrow Z^4 l_{11}^{3/2} + 2\pi i R \quad (3.4)$$

is an invariance of the theory, or by decomposing $Z^4 = X^4 + iX^8$ that X^8 is compactified with radius R . This is the radius of the M-theory circle. By letting $\frac{v}{k}l_{11}^{3/2} = R \rightarrow 0$ one recovers the theory of D2-branes of type IIA string theory in flat space as the direction

⁸Here the fields X^A after higgsing are $N \times N$ hermitian matrices which will be expanded in a basis consisting of the unit matrix and a hermitian set of $SU(N)$ generators.

⁹The trace parts of the field theory scalars are related to spacetime coordinates by multiplication with a factor of $l_{11}^{3/2}$.

X^8 shrinks to zero. At low energies this corresponds to an $\mathcal{N} = 8$ SYM theory in 2+1 dimensions, which involves seven, as opposed to eight, scalars. As a result it is natural to expect that the Goldstone modes in the Higgs mechanism, rendering the gauge fields dynamical, should be precisely X^8 , at least for the diagonal part X_0^8 of the general $N \times N$ matrix, corresponding to the center of mass motion of the branes. On the other hand for the BLG-model studied in [2],¹⁰ which employs real fields, the scalar degree of freedom that disappeared was exactly the one that developed the VEV, as it should due to it not having a complex partner. For our particular choice of VEV this would correspond to X_a^4 and X_0^4 being singled out as opposed to X_0^8 implied by (3.4). This is a sign that the orbifold picture is not an appropriate dual description of the $SU(N) \times SU(N)$ theory, which is in line with the fact that there is no known string theory interpretation for ABJM with this choice of gauge group. In fact, we will see that in the $U(N) \times U(N)$ case the Goldstone modes that disappear are X_a^4 and X_0^8 , exactly as needed to satisfy both previous pictures.

The pure ABJM lagrangian for both $SU(N) \times SU(N)$ and $U(N) \times U(N)$ gauge groups is given by [34]:

$$S = \int d^3x \left[\frac{k}{4\pi} \varepsilon^{\mu\nu\lambda} \text{tr} \left(A_\mu^L \partial_\nu A_\lambda^L + \frac{2i}{3} A_\mu^L A_\nu^L A_\lambda^L - A_\mu^R \partial_\nu A_\lambda^R - \frac{2i}{3} A_\mu^R A_\nu^R A_\lambda^R \right) - \text{tr} \hat{D}_\mu Z_A^\dagger \hat{D}^\mu Z^A - i \text{tr} \bar{\Psi}^{\dagger A} \gamma^\mu \Psi \hat{D}_\mu \Psi_A - V_{bos} - V_{Yukawa} \right], \quad (3.5)$$

where

$$V_{bos} = -\frac{4\pi^2}{3k^2} \text{tr} \left(Z^A Z_A^\dagger Z^B Z_B^\dagger Z^C Z_C^\dagger + Z_A^\dagger Z^A Z_B^\dagger Z^B Z_C^\dagger Z^C + 4Z^A Z_B^\dagger Z^C Z_A^\dagger Z^B Z_C^\dagger - 6Z_A^\dagger Z^B Z_B^\dagger Z^A Z_C^\dagger Z^C \right) \quad (3.6)$$

$$V_{Yukawa} = -\frac{2\pi i}{k} \text{tr} \left(Z_A^\dagger Z^A \bar{\Psi}^{\dagger B} \Psi_B - \bar{\Psi}^{\dagger B} Z^A Z_A^\dagger \Psi_B - 2Z_A^\dagger Z^B \bar{\Psi}^{\dagger A} \Psi_B + 2\bar{\Psi}^{\dagger B} Z^A Z_B^\dagger \Psi_A - \epsilon^{ABCD} Z_A^\dagger \bar{\Psi}_B Z_C^\dagger \Psi_D + \epsilon^{ABCD} Z^A \bar{\Psi}^{\dagger B} Z^C \Psi^{\dagger D} \right) \quad (3.7)$$

and

$$\hat{D}_\mu Z^A = \partial_\mu Z^A + iA_\mu^L Z^A - iZ^A A_\mu^R. \quad (3.8)$$

For the higgsing it is appropriate to define

$$A_\mu^+ = \frac{1}{2}(A_\mu^L + A_\mu^R), \quad A_\mu^- = \frac{1}{2}(A_\mu^L - A_\mu^R), \quad (3.9)$$

which, observing that $A_\mu^L Z^A - Z^A A_\mu^R = [A_\mu^+, Z^A] + \{A_\mu^-, Z^A\}$, translates into

$$\begin{aligned} \hat{D}_\mu Z^A &= D_\mu Z^A + i\{A_\mu^-, Z^A\} \\ D_\mu Z^A &= \partial_\mu Z^A + i[A_\mu^+, Z^A] \\ F_{\mu\nu}^+ &= \partial_\mu A_\nu^+ - \partial_\nu A_\mu^+ + i[A_\mu^+, A_\nu^+]. \end{aligned} \quad (3.10)$$

Note that the abelian gauge fields do not appear in the covariant derivative D_μ . In terms of these new variables the CS part of the lagrangian becomes

$$S_{CS} = \int d^3x \frac{k}{2\pi} \varepsilon^{\mu\nu\lambda} \text{tr} \left(A_\mu^- F_{\nu\lambda}^+ + \frac{2i}{3} A_\mu^- A_\nu^- A_\lambda^- \right). \quad (3.11)$$

The analysis will be performed for the bosonic sector but the fermions follow suit.

¹⁰That case corresponds to the $SU(N) \times SU(N)$ model for $N = 2$.

3.1.1 The $SU(N) \times SU(N)$ case

We start by looking at the BLG case in Van Raamsdonk's $SU(2) \times SU(2)$ formulation, before generalizing to $SU(N) \times SU(N)$. The identification between the three-algebra and the conventional gauge theory formulations is (adopting the notation of [32] to that used in this paper)

$$\begin{aligned} X^I &= X_0^I + i\sigma_i X_i^I, \quad \Psi^I = \psi_0^I + i\sigma_i \psi_i^I \\ A_\mu^L &= A_{\mu 4i}^L \sigma_i, \quad A_\mu^R = A_{\mu 4i}^R \sigma_i, \end{aligned} \quad (3.12)$$

with

$$\begin{aligned} A_{\mu ab} &= -\frac{1}{2f}(A_{\mu ab}^L + A_{\mu ab}^R), \\ A_{\mu ab}^{L(R)} &= \pm \frac{1}{2} \epsilon_{abcd} A_{\mu cd}^{L(R)} \end{aligned} \quad (3.13)$$

and the three-algebra structure constants $f^{abcd} = f\epsilon^{abcd}$, while the $SU(2)$ generators are normalized by $\text{tr}(\sigma_i \sigma_j) = 2\delta_{ij}$. Note that we choose the old and new covariant derivatives to differ by a factor of i . Still in real notation ($I = 1, \dots, 8$) we thus make the identification:

$$\begin{aligned} D_\mu X^{Ia} &= \partial_\mu X^{Ia} + f^{abc} A_{\mu bc} X^{Id} \\ D_\mu X^I &= \partial_\mu X^I + iA_\mu^L X^I - iX^I A_\mu^R. \end{aligned} \quad (3.14)$$

The generalization to $SU(N) \times SU(N)$ was given in [8]. However, there it was applied to the $U(N) \times U(N)$ ABJM case although the $U(1)$ factors seem not to have been included.

In order to proceed in the general $SU(N) \times SU(N)$ case one has to expand the complex ABJM scalars as follows:¹¹

$$Z^A = z_0^A T^0 + i z_a^A T^a; \quad \Psi^A = \psi_0^A T^0 + i \psi_a^A T^a; \quad A_\mu^R = A_\mu^{aR} T_a; \quad A_\mu^L = A_\mu^{aL} T_a \quad (3.15)$$

and define the A_μ^\pm as above. Then one can split the fields into real and imaginary parts, while giving a VEV to the real scalar component along $T^0 = \mathbb{1}_{N \times N}$.¹²

$$\begin{aligned} Z^A &= \left(\frac{X_0^A}{\sqrt{2}} + v\delta^{A,4} \right) T^0 + i \frac{X_0^{A+4}}{\sqrt{2}} T^0 + i \frac{X_a^A}{\sqrt{2}} T^a - \frac{X_a^{A+4}}{\sqrt{2}} T^a \\ \Psi^A &= \psi_0^A T^0 + i \psi_a^A T^a + i \psi_0^{A+4} T^0 - \psi_a^{A+4} T^a. \end{aligned} \quad (3.16)$$

With the definitions $[T^a, T^b] = i f^{ab}{}_c T^c$, $\{T^a, T^b\} = d^{ab}{}_c T^c$ one gets

$$\begin{aligned} \hat{D}_\mu Z^A &= \frac{\partial_\mu X_0^A}{\sqrt{2}} T^0 - \frac{D_\mu X_a^{A+4}}{\sqrt{2}} T^a + \frac{i \partial_\mu X_0^{A+4}}{\sqrt{2}} T^0 + \frac{i D_\mu X_a^A}{\sqrt{2}} T^a \\ &\quad + 2iv A_{\mu a}^- T^a \delta^{A4} + i \sqrt{2} A_{\mu a}^- X_0^A T^a - \sqrt{2} A_{\mu a}^- X_0^{A+4} T^a \\ &\quad - \frac{i}{\sqrt{2}} A_\mu^{-a} d_{abc} T^c X_b^{A+4} - \frac{1}{\sqrt{2}} A_\mu^{-a} d_{abc} T^c X_b^A \\ &\quad + i[v(B_{\mu 4}^A + q C_\mu \delta^{A4}) + \text{subleading}], \end{aligned} \quad (3.17)$$

¹¹We normalize the $SU(N)$ generators as $\text{tr}(T^a T^b) = \delta^{ab}$, while $T^0 = \mathbb{1}_{N \times N}$.

¹²Note that compared to [8] we use a real and not an imaginary VEV. As a result the roles of X^4 and X^8 will be interchanged in the final result compared to that reference.

where we note that only the first of the A_μ^- terms involves the VEV v . Note also that in the last line we wrote also the B and C terms in the covariant derivative, since they combine with the rest. Indeed, we see that $[v(B_{\mu 4}^4 + qC_\mu) + \frac{1}{\sqrt{2}}\partial_\mu X_0^8]$, $[v\text{Re}(B_{\mu 4}^{A'}) + \frac{1}{\sqrt{2}}\partial_\mu X_0^{A'+4}]$ and $[-v\text{Im}(B_{\mu 4}^{A'}) + \frac{1}{\sqrt{2}}\partial_\mu X_0^{A'}]$ appear together signalling the Higgs mechanism, as we will discuss in more detail in a later section. However, since in this subsection we look only at the pure ABJM part we will ignore B and C in the following discussion. Then

$$\text{tr} |\hat{D}_\mu Z^A|^2 = N \frac{(\partial_\mu X_0^A)^2}{2} + N \frac{(\partial_\mu X_0^{A+4})^2}{2} + \left(\frac{(D_\mu X)_c^{A+4}}{\sqrt{2}} \right)^2 + \left(2v A_{\mu c}^- \delta^{A4} + \frac{(D_\mu X)_c^A}{\sqrt{2}} \right)^2 + \text{subleading}, \quad (3.18)$$

where all the terms with A_μ^- , except the one shown explicitly, are subleading in the large v limit. Adding for free a term that gives zero under the integral by the Bianchi identity

$$\frac{k}{2\pi} \varepsilon^{\mu\nu\lambda} \frac{1}{v} \frac{1}{2\sqrt{2}} (D_\mu X)_a^4 F_{\nu\lambda}^{+a} \quad (3.19)$$

we get, together with the CS terms,

$$\begin{aligned} S &= \int d^3x \left[\frac{k}{2\pi} \varepsilon^{\mu\nu\lambda} \text{tr} \left(A_\mu^- F_{\nu\lambda}^+ + \frac{2i}{3} A_\mu^- A_\nu^- A_\lambda^- \right) - \text{tr} |\hat{D}_\mu Z^A|^2 \right] \\ &= \int d^3x \left[\frac{k}{2\pi} \varepsilon^{\mu\nu\lambda} (A_{\mu a}^- + \frac{1}{2v} \frac{1}{\sqrt{2}} (D_\mu X)_a^4) F_{\nu\lambda a}^+ - (2v A_{\mu a}^- + \frac{1}{\sqrt{2}} (D_\mu X)_a^4)^2 \right. \\ &\quad \left. - \frac{1}{2} (D_\mu X)_a^{I'} (D^\mu X)_a^{I'} - \frac{1}{2} N D_\mu X_0^I D^\mu X_0^I + \text{higher order} \right], \quad (3.20) \end{aligned}$$

where $I = 1, \dots, 8$ and $I' = \{I \neq 4\}$. The higher order terms also include a contribution proportional to $(A_\mu^-)^3$. However, these terms are subleading in the limit $k, v \rightarrow \infty$ and can hence be ignored.

Now one can perform a shift involving X_a^4 :

$$A_{a\mu}^- \rightarrow A_{a\mu}^- - \frac{1}{2v} \frac{1}{\sqrt{2}} D_\mu X_a^4 \quad (3.21)$$

that leads to the simple expression

$$\begin{aligned} S &= \int d^3x \left(\frac{k}{2\pi} \varepsilon^{\mu\nu\lambda} A_{\mu a}^- F_{\nu\lambda a}^+ - 4v^2 A_{\mu a}^- A_a^{-\mu} - \frac{1}{2} (D_\mu X)_a^{I'} (D^\mu X)_a^{I'} \right. \\ &\quad \left. - \frac{1}{2} N \partial_\mu X_0^I \partial^\mu X_0^I + \text{higher order} \right). \quad (3.22) \end{aligned}$$

In the above the X_a^4 component has vanished. This is the Goldstone mode that will render the $\text{SU}(N)$ gauge field $A_{\mu a}^+$ dynamical. Note that the X_0 's are decoupled.

We finally integrate out $A_{\mu a}^-$ to obtain

$$A_{\mu a}^- = \frac{k}{16\pi v^2} \varepsilon_{\mu\nu\lambda} F_a^{+\nu\lambda} + \text{higher order} \quad (3.23)$$

and upon plugging this into the above expression for the action we get

$$S = \int d^3x \left(-\frac{k^2}{32\pi^2 v^2} F^{+\mu\nu a} F_{\mu\nu}^{+a} - \frac{1}{2} (D_\mu X)_a^{I'} (D^\mu X)_a^{I'} - \frac{1}{2} N \partial_\mu X_0^I \partial^\mu X_0^I + \text{higher order} \right). \quad (3.24)$$

Then using the definition

$$\frac{k^2}{32\pi^2 v^2} = \frac{1}{4g_{YM}^2} \quad (3.25)$$

and taking the limit $k, v \rightarrow \infty$, with $\frac{k}{v} = \text{fixed}$, the higher order terms drop out. Furthermore, performing an abelian duality transformation on the free scalar degree of freedom X_0^4 , one recovers an additional U(1) that combines with the SU(N) factor. The result is nothing but the bosonic part for the kinetic terms of U(N) SYM theory in three dimensions, including all the correct coefficients.

The situation can be summarized by referring to the coset $\text{SU}(N) \times \text{SU}(N)/\text{SU}(N)$ where the group in the denominator is the diagonal subgroup of those in the numerator. By standard Higgs arguments one can use the elements in the coset to rotate *e.g.* the $N \times N$ matrix X^4 into its diagonal part X_0^4 . Due to the symmetry of the lagrangian the corresponding gauge transformation will trivially cancel out in all non-kinetic terms proving that X_a^4 will not appear anywhere in those terms. This will be explicitly demonstrated later in this section. What remains to be analyzed is the kinetic terms which were dealt with above. We now turn to the $\text{U}(N) \times \text{U}(N)$ case where also the U(1) factors will experience a Higgs effect.

3.1.2 The $\text{U}(N) \times \text{U}(N)$ case

In this case, which is the one that is relevant for the physics of M2-branes, we will obtain some important differences compared to the previous result.

The expansion of the fields now includes the U(1) factors for the gauge fields

$$\begin{aligned} Z^A &= z_0^A T^0 + i z_a^A T^a, \quad \Psi^A = \psi_0^A T^0 + i \psi_a^A T^a, \\ A_\mu^L &= A_\mu^{L0} T^0 + A_\mu^{La} T^a, \quad A_\mu^R = A_\mu^{R0} T^0 + A_\mu^{Ra} T^a, \end{aligned} \quad (3.26)$$

and

$$\begin{aligned} Z^A &= \left(\frac{X_0^A}{\sqrt{2}} + v \delta^{A,4} \right) T^0 + i \frac{X_0^{A+4}}{\sqrt{2}} T^0 + i \frac{X_a^A}{\sqrt{2}} T^a - \frac{X_a^{A+4}}{\sqrt{2}} T^a, \\ \Psi^A &= \psi_0^A T^0 + i \psi_a^A T^a + i \psi_0^{A+4} T^0 - \psi_a^{A+4} T^a, \end{aligned} \quad (3.27)$$

with the rest of the definitions remaining as for $\text{SU}(N) \times \text{SU}(N)$. As a result one gets some extra terms involving $A_{\mu 0}^-$ in

$$\begin{aligned} \hat{D}_\mu Z^A &= \frac{\partial_\mu X_0^A}{\sqrt{2}} T^0 - \frac{D_\mu X_a^{A+4}}{\sqrt{2}} T^a + \frac{i \partial_\mu X_0^{A+4}}{\sqrt{2}} T^0 + \frac{i D_\mu X_a^A}{\sqrt{2}} T^a \\ &\quad + 2i v A_{\mu a}^- T^a \delta^{A4} + i \sqrt{2} A_{\mu a}^- X_0^A T^a - \sqrt{2} A_{\mu a}^- X_0^{A+4} T^a + A_\mu^{-a} d_{abc} T^c X_b^{A+4} \\ &\quad - i A_\mu^{-a} d_{abc} T^c X_b^A + 2i v A_{\mu 0}^- T^0 \delta^{A4} + i \sqrt{2} A_{\mu 0}^- X_0^A T^0 - \sqrt{2} A_{\mu 0}^- X_0^{A+4} T^0 \\ &\quad - \sqrt{2} A_{\mu 0}^- X_a^A T^a - i \sqrt{2} A_{\mu 0}^- X_a^{A+4} T^a + i [v (B_{\mu 4}^A + q C_\mu \delta^{A4}) + \text{subleading}] . \end{aligned} \quad (3.28)$$

As in the $\text{SU}(N) \times \text{SU}(N)$ case, we introduced the B and C fields since they combine with $X_0^{A'}$, $X_0^{A'+4}$, X_0^8 and now additionally with $A_{0\mu}^-$. Specifically, we have $[v (B_{\mu 4}^A + q C_\mu + A_{0\mu}^-) +$

$\frac{1}{\sqrt{2}}\partial_\mu X_0^8]$, $[v\text{Re}(B_\mu^{A'} + \frac{1}{\sqrt{2}}\partial_\mu X_0^{A'+4})]$ and $[-v\text{Im}(B_\mu^{A'} + \frac{1}{\sqrt{2}}\partial_\mu X_0^{A'})]$ appearing together, once again signalling the Higgs mechanism. However the B, C fields will be dropped for the moment and dealt with in later subsections. Then we obtain

$$\begin{aligned} \text{tr} |\hat{D}_\mu Z^A|^2 &= N \frac{(\partial_\mu X_0^A)^2}{2} + \left(\frac{(D_\mu X)_c^{A+4}}{\sqrt{2}} \right)^2 + \left(2v A_{\mu c}^- \delta^{A4} + \frac{(D_\mu X)_c^A}{\sqrt{2}} \right)^2 \\ &+ N \left(\frac{1}{\sqrt{2}} \partial_\mu X_0^{A+4} + 2v A_{\mu 0}^- \delta^{A4} \right)^2 + \text{subleading} . \end{aligned} \quad (3.29)$$

Adding for free the two terms, of which the second is new compared to the $\text{SU}(N) \times \text{SU}(N)$ case,

$$\frac{k}{2\pi} \varepsilon^{\mu\nu\lambda} \frac{1}{v} \frac{1}{2\sqrt{2}} (D_\mu X)_a^4 F_{\nu\lambda}^{+a} + \frac{Nk}{2\pi} \varepsilon^{\mu\nu\lambda} \frac{1}{v} \frac{1}{2\sqrt{2}} (\partial_\mu X_0^8) F_{\nu\lambda}^{+0} \quad (3.30)$$

and with the inclusion of the CS terms, we get that the lagrangian is

$$\begin{aligned} S &= \int d^3x \left[\frac{k}{2\pi} \varepsilon^{\mu\nu\lambda} \text{tr} \left(A_\mu^- F_{\nu\lambda}^+ + \frac{2i}{3} A_\mu^- A_\nu^- A_\lambda^- \right) - \text{tr} |\hat{D}_\mu Z^A|^2 \right] \\ &= \int d^3x \left[\frac{k}{2\pi} \varepsilon^{\mu\nu\lambda} \left(A_{\mu a}^- + \frac{1}{2v} \frac{1}{\sqrt{2}} (D_\mu X)_a^4 \right) F_{\nu\lambda}^{+a} + \frac{Nk}{2\pi} \varepsilon^{\mu\nu\lambda} \left(A_{\mu 0}^- + \frac{1}{2v} \frac{1}{\sqrt{2}} (\partial_\mu X)_0^8 \right) F_{\nu\lambda}^{+0} \right. \\ &\quad - \left(2v A_{\mu a}^- + \frac{1}{\sqrt{2}} (D_\mu X)_a^4 \right)^2 - N \left(\frac{1}{\sqrt{2}} \partial_\mu X_0^{A+4} + 2v A_{\mu 0}^- \delta^{A4} \right)^2 \\ &\quad \left. - \frac{1}{2} (D_\mu X)_a^{I'} (D^\mu X)_a^{I'} - \frac{1}{2} N \partial_\mu X_0^A \partial^\mu X_0^A + \text{higher order} \right] , \end{aligned} \quad (3.31)$$

where $I' = \{I \neq 4\}$. The higher order terms will once again include $(A_\mu^-)^3$, which will lead to subleading expressions in the limit $k, v \rightarrow \infty$.

Note that in this case we can perform a double shift; both the $A_{\mu a}^-$ shift of (3.21) as well as one for the new abelian component $A_{\mu 0}^-$:

$$A_{\mu a}^- \rightarrow A_{\mu a}^- - \frac{1}{2v} \frac{1}{\sqrt{2}} (D_\mu X)_a^4 \quad \text{and} \quad A_{\mu 0}^- \rightarrow A_{\mu 0}^- - \frac{1}{2v} \frac{1}{\sqrt{2}} (\partial_\mu X_0^8) \quad (3.32)$$

and that leads to

$$\begin{aligned} S &= \int d^3x \left(\frac{k}{2\pi} \varepsilon^{\mu\nu\lambda} (A_{\mu a}^- F_{\nu\lambda}^{a+} + N A_{\mu 0}^- F_{\nu\lambda}^{+0}) - 4v^2 A_{\mu a}^- A_a^{-\mu} - 4Nv^2 A_{\mu 0}^- A_0^{-\mu} \right. \\ &\quad \left. - \frac{1}{2} (D_\mu X)_a^{I'} (D^\mu X)_a^{I'} - \frac{1}{2} N \partial_\mu X_0^{\tilde{I}'} \partial^\mu X_0^{\tilde{I}'} + \text{higher order} \right) , \end{aligned} \quad (3.33)$$

where $\tilde{I}' = \{I \neq 8\}$.

It is important to observe that in the above both the X_a^4 and the X_0^8 components vanish to leading order, as opposed to only X_a^4 for $\text{SU}(N) \times \text{SU}(N)$. These fields make up the Goldstone modes that render, respectively, $A_{\mu a}^+$ and $A_{\mu 0}^+$ dynamical to give back a $\text{U}(N)$ gauge field. This situation corresponds to the coset $\text{U}(N) \times \text{U}(N)/\text{U}(N)$. Without the vanishing of the X_0^8 one would have ended up with excessive degrees of freedom compared to the initial theory. This result is also in line with the spacetime orbifold picture, as it should apply to the trace component X_0^8 , as opposed to the non-trace X_a^4 .

We can now integrate A_μ^- out to obtain

$$A_\mu^- = \frac{k}{16\pi v^2} \varepsilon_{\mu\nu\lambda} F^{+\nu\lambda} + \text{higher order} \quad (3.34)$$

and upon plugging into the above this gives rise to

$$S = \int d^3x \left[\text{tr} \left(-\frac{k^2}{32\pi^2 v^2} F^{+\mu\nu} F_{\mu\nu}^+ \right) - \frac{1}{2} (D_\mu X)_a^{I'} (D^\mu X)_a^{I'} - \frac{1}{2} N \partial_\mu X_0^{\tilde{I}'} \partial^\mu X_0^{\tilde{I}'} + \text{higher order} \right]. \quad (3.35)$$

As for $\text{SU}(N) \times \text{SU}(N)$, along with the definition (3.25) for the YM coupling and taking the limit $k, v \rightarrow \infty$, with $\frac{v}{k} \rightarrow \text{fixed}$, the higher order terms drop out and upon combining the remaining traceless part of X_a^8 with the trace part of X_0^4 this is nothing but the bosonic part for the kinetic terms of $\text{U}(N)$, three-dimensional SYM theory.

Similarly to the discussion at the end of the previous subsection, the Higgs phenomenon can in this case be related to the coset $\text{U}(N) \times \text{U}(N)/\text{U}(N)$ where global factors in the definition of $\text{U}(N)$ are not considered. Thus a full $N \times N$ matrix is involved in the higgsing which we saw above corresponded to X_0^8 and X_a^4 . When gravity is added to this picture the situation becomes a bit more interesting, as we will discuss next.

3.2 Higgsing including the gravity sector

We will next investigate what happens when the gravitational sector is added to the theory. Explicitly, the bosonic subsector of the lagrangian relevant for this analysis is¹³

$$L = g_M^{-2} L_{CS(\tilde{\omega})} - e \partial Z^A \partial \bar{Z}_A - \frac{e}{8} R |Z|^2 - e V_{pot}(Z, \bar{Z}), \quad (3.36)$$

where V_{pot} is the full bosonic potential. We remind the reader that apart from the one-trace term coming from the original ABJM lagrangian the potential also contains double- and triple-trace terms arising from the topological gauging. The terms in the potential are scaled in such a way that they pick up one factor of g_M^2 for each trace beyond the first, following the relevant discussion in Section 2.

We note that due to the gauge-gravity interaction terms the introduction of the VEV, $Z^A = v \delta^{A4} + z^A$, leads to a nonzero cosmological term in the lagrangian

$$V_{pot}(v) = V_{pot}^{3-\text{trace}}(v) = \frac{1}{4^4} N^3 g_M^4 v^6. \quad (3.37)$$

It is also immediately clear that the third term in the above lagrangian will lead to an Einstein-Hilbert term that will play an important role later.

¹³The other set of limits discussed in this paper in Section 4 are based on the same lagrangian but with an extra overall factor of g_M^2 .

3.2.1 Background: solution, decoupling limit

From (3.36) one obtains the following equations of motion, by varying with respect to the scalars and the metric respectively:

$$\begin{aligned} & \square Z^D - \frac{1}{8} R Z^D + \frac{5}{4 \cdot 64} g_M^4 |Z|^4 Z^D - \frac{g_M^4}{32} Z^D \text{tr}(Z^A Z_C^\dagger) \text{tr}(Z^C Z_A^\dagger) \\ & - \frac{g_M^4}{16} |Z|^2 Z^A \text{tr}(Z^D Z_A^\dagger) + \frac{g_M^4}{16} Z^A \text{tr}(Z^B Z_A^\dagger) \text{tr}(Z^D Z_B^\dagger) = 0 \end{aligned} \quad (3.38)$$

and

$$\begin{aligned} & -e(\partial_\mu Z^A \partial_\nu \bar{Z}_A - \frac{1}{2} g_{\mu\nu} \partial^\rho Z^A \partial_\rho \bar{Z}_A) + g_M^{-2} \mathcal{C}_{\mu\nu} + \frac{e}{2} g_{\mu\nu} V_{pot} \\ & - \frac{e}{8} (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) |Z|^2 + \frac{e}{8} (\nabla_\mu \nabla_\nu |Z|^2 - g_{\mu\nu} \square |Z|^2) = 0, \end{aligned} \quad (3.39)$$

where

$$\mathcal{C}_{\mu\nu} \equiv \epsilon_\mu^{\alpha\beta} \nabla_\alpha (R_{\beta\nu} - \frac{1}{4} g_{\beta\nu} R) \quad (3.40)$$

is the Cotton tensor.

It is now straightforward to verify that a constant VEV v solves the scalar equations of motion: The $D = 1, 2, 3$ and the imaginary part of the $D = 4$ equation are satisfied by the VEV, while the real part of the $D = 4$ equation gives

$$R^{BG} = -\frac{3}{32} (g_M v \sqrt{N})^4, \quad (3.41)$$

which suggests an AdS background geometry. Although this looks like an Einstein equation, it comes from the *scalar* equations of motion, so *a priori* it is a constraint on top of the graviton field equations which we next check for consistency.

For AdS solutions the Cotton tensor vanishes, since the geometry is conformal, reducing the gravitational field equation to

$$\frac{1}{4} (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R)|_{AdS} N v^2 = V_{pot}(v) g_{\mu\nu}, \quad (3.42)$$

which in turn gives

$$R_{\mu\nu}^{BG} = 2\Lambda g_{\mu\nu} \quad \text{with} \quad \Lambda = -\frac{1}{64} (g_M v \sqrt{N})^4. \quad (3.43)$$

The trace gives exactly (3.41), as expected for AdS₃ with $R = 6\Lambda$.

Since one should be able to recover the pure ABJM lagrangian for *generic* values of the ABJM fields and parameters but *small perturbations* around the (AdS) gravitational background, we would next like to understand whether it is possible to decouple gravity from the gauge theory in the lagrangian (2.20)-(2.28) using the gravitational coupling g_M .

This decoupling limit is in fact $g_M \rightarrow 0$ with all other parameters fixed. In this limit the terms in the lagrangian that couple gravity to matter become weak and the background becomes flat, as seen from (3.41). The double-trace and triple-trace interaction terms in (2.28) vanish trivially. Since the $R^{BG}|Z|^2$ background term vanishes, and we are

only considering perturbations much smaller than the background, $R|Z|^2$ also vanishes in the limit.¹⁴ The remaining single-trace interaction terms involve at least one gravitino. By rescaling the Rarita-Schwinger action to the canonical form through multiplying the gravitini by g_M cancelling the $\frac{1}{g_M^2}$ coefficient, all the single-trace interaction terms pick up g_M factors and vanish as well. Therefore $g_M \rightarrow 0$ decouples gravity completely and we are left with the pure ABJM lagrangian in a flat background.

In summary, the VEV is indeed a consistent solution of the equations of motion, giving an anti-de Sitter space as a background. The limit $g_M \rightarrow 0$, with all other parameters fixed and for small perturbations around the background, decouples gravity from the ABJM part of the action.

3.2.2 Higgsed supergravity action and the chiral point

We now write down the relevant bosonic part of the higgsed gravitational lagrangian, keeping in mind that this is in the presence of the AdS_3 background we have just found. This can be readily determined and reads

$$L_{sugra} = -e \frac{v^2 N}{8} (R - 2\Lambda) + \frac{1}{2g_M^2} \epsilon^{\mu\nu\rho} \text{Tr}_\alpha (\tilde{\omega}_\mu \partial_\nu \tilde{\omega}_\rho + \frac{2}{3} \tilde{\omega}_\mu \tilde{\omega}_\nu \tilde{\omega}_\rho) . \quad (3.44)$$

In order for it to be compatible with the ABJM part of the higgsing process the above is valid in the limit $v \rightarrow \infty, \lambda \rightarrow 0$, with $v\lambda = \text{fixed}$. However, in the presence of gravity one needs to additionally supplement the former with a condition on their scalings relative to the coupling g_M . This can be seen as follows: In Eq. (3.41) we discovered that the curvature for the background solution was given by

$$R^{BG} = -\frac{3}{32} g_M^4 v^4 N^2 .$$

For a background space with fixed and finite curvature, the RHS of this equation must not scale with v . This can be achieved by letting $g_M \rightarrow 0$ in such a way that

$$\bar{g}_M^2 = g_M^2 v^2 N = \text{fixed} . \quad (3.45)$$

For the background this means that, since $R = 6\Lambda$,

$$\Lambda = -\frac{1}{64} \bar{g}_M^4 . \quad (3.46)$$

The Newton constant can also be read off from the above lagrangian:

$$\kappa^2 = \frac{8}{v^2 N} . \quad (3.47)$$

¹⁴We note however that it was essential that we only considered small perturbations around the background, otherwise for generic perturbations and generic nonzero VEV the $R|Z|^2$ term gives gravity with finite gravitational coupling $\kappa^2 \sim \frac{1}{v}$.

If we then adopt the definition of the constant μ in front of the gravitational CS-term as done in the work of Li, Song and Strominger [11] we have $\kappa^2\mu = g_M^2$ and hence

$$\mu = \frac{\bar{g}_M^2}{8}. \quad (3.48)$$

Finally, expressing the cosmological constant Λ in terms of a length ℓ as $\Lambda = -\frac{1}{\ell^2}$ one finds that¹⁵

$$\mu\ell = 1, \quad (3.49)$$

i.e., as was first noticed in [9], the theory sits at a ‘chiral point’ in the sense of [11]. Note, however, that the signs of the Einstein-Hilbert term and the cosmological term in (3.44) are opposite to the ones used in [11], although they coincide with the ones normally used in TMG. We will come back to this point in Section 4.1.

The limit with $\bar{g}_M^2 = \text{fixed}$ looks similar to the weak gravity limit discussed at the end of the previous section although now one has $v \rightarrow \infty$, $g_M \rightarrow 0$, as well as a nontrivial gravitational background. As a result it is not the same as the trivial limit that decouples gravity and gives back pure ABJM. In fact, we will next find that at the end of the higgsing process one obtains finite interaction terms between gravity and ABJM in addition to the finite curvature gravitational piece. We will however postpone a full discussion of the physics in the gravity sector post-higgsing to Section 4.

3.3 The interaction terms

We now turn to the interaction terms of the full action. These do not show any qualitative differences between $\text{SU}(N) \times \text{SU}(N)$ and $\text{U}(N) \times \text{U}(N)$ so for the remainder of this section we will assume the latter, which is the case of direct physical interest.

For the initial purpose of finding the leading contribution in v we can refrain from using the full expansion (3.16), as this is rather involved and in fact some of the terms might end up canceling. Instead we first perform the expansion around the VEV v using the expression

$$Z^A = v\delta^{A,4} + z^A \quad (3.50)$$

and derive the result for all the interaction terms as an order-by-order expansion in v .

3.3.1 Bosonic terms

The bosonic terms can be organized as single-, double- and triple-trace. The single-trace 6-scalar structures appear only in the ABJM part of the potential V_6 ,

$$\begin{aligned} -V_6 = & \text{tr}(Z^A Z_Z^\dagger Z^B Z_B^\dagger Z^C Z_C^\dagger + Z_Z^\dagger Z^A Z_B^\dagger Z^B Z_C^\dagger Z^C \\ & + 4Z^A Z_B^\dagger Z^C Z_A^\dagger Z^B Z_C^\dagger - 6Z^A Z_B^\dagger Z^B Z_A^\dagger Z^C Z_C^\dagger), \end{aligned} \quad (3.51)$$

¹⁵Here we define ℓ to be positive. The sign of ℓ does not enter the bosonic part of the theory, but is related to the chirality of the supersymmetries when the fermionic sector is included.

which was already analyzed in [8]. Here we simply wish to observe that all the terms scaling like v^6, \dots, v^3 vanish and that one is left with a potential which is of order $v^2 \lambda^2 \propto g_{YM}^2$, and hence fourth order in the scalar fields, as expected [8]. The remaining terms are subleading in v and will vanish in the $v \rightarrow \infty$ limit. The term surviving in the limit is

$$-V_6 \rightarrow -\frac{g_{YM}^2}{4} \text{tr} \left([X^{I'}, X^{J'}] [X^{J'}, X^{I'}] \right) \quad (3.52)$$

but since this term is part of the ordinary Yang-Mills theory it will not be mentioned further when the scalar field interaction terms are discussed below.

For the rest of the interaction terms the results can be obtained using the formulas from Appendix A. For the double-trace expressions we find that the v^6, v^5 and v^4 terms are identically zero, while for lower orders we obtain

$$\begin{aligned} -V_{2\text{-trace}} &= g_M^2 \frac{\lambda}{8} |Z|^2 \text{tr}(Z_D^\dagger Z^C Z_C^\dagger Z^D - Z_D^\dagger Z^D Z_C^\dagger Z^C) \\ &\quad + g_M^2 \frac{\lambda}{2} \text{tr}(Z^D Z_B^\dagger) \text{tr}(Z_D^\dagger Z^B Z_C^\dagger Z^C - Z_D^\dagger Z^C Z_C^\dagger Z^B) \\ &= g_M^2 \lambda \left[\frac{1}{4} v^3 N \text{tr}((z^4 + z_4^\dagger)[z_D^\dagger, z^D]) + \frac{1}{2} v^3 N \text{tr}((z^4 + z_4^\dagger)[z_4^\dagger, z^4]) \right. \\ &\quad - \frac{v^2 N}{8} \text{tr}(z_D^\dagger z^D z_C^\dagger z^C - z^D z_D^\dagger z^C z_C^\dagger) + \frac{v^2 N}{2} \text{tr}(z_4^\dagger z^4 z_C^\dagger z^C - z^4 z_4^\dagger z^C z_C^\dagger) \\ &\quad + \frac{v^2}{4} \text{tr}(z^4 - z_4^\dagger) \text{tr}((z^4 - z_4^\dagger)[z_D^\dagger, z^D]) + \frac{v^2}{2} \text{tr} z^D \text{tr}(z_D^\dagger [z_C^\dagger, z^C] + z_D^\dagger [z^4, z_4^\dagger]) \\ &\quad \left. + \frac{v^2}{2} \text{tr} z_D^\dagger \text{tr}(z^D [z_C^\dagger, z^C] + z^D [z^4, z_4^\dagger]) \right] + \mathcal{O}(v). \end{aligned} \quad (3.53)$$

Here we should note that the coefficients for the leading terms in v are $\sim g_M^2 \lambda v^3$ which is just $\bar{g}_M^2 g_{YM}$ and thus fixed in the scaling limit. As in the previous case, any remaining contributions are of lower order in v and will vanish as $v \rightarrow \infty$.

For the triple-trace part the situation is more intricate. In this case no terms will vanish identically and, due to the special properties of the set of limits used here, we must keep not only the terms that are multiplied by parameters fixed under this scaling but also terms that blow up. These latter terms will determine the background geometry and are crucial for the interpretation of the results. We find

$$\begin{aligned} -V_{3\text{-trace}} &= g_M^4 \left[\frac{5}{12 \cdot 64} |Z|^6 - \frac{1}{32} |Z|^2 \text{tr}(Z^A Z_C^\dagger) \text{tr}(Z^C Z_A^\dagger) \right. \\ &\quad \left. + \frac{1}{48} \text{tr}(Z^A Z_C^\dagger) \text{tr}(Z^B Z_A^\dagger) \text{tr}(Z^C Z_B^\dagger) \right] \\ &= g_M^4 \left\{ -\frac{1}{256} (v^2 N + v \text{tr}(z^4 + z_4^\dagger))^3 - \frac{3}{256} \text{tr}(z^A z_A^\dagger) (v^2 N + v \text{tr}(z^4 + z_4^\dagger))^2 \right. \\ &\quad + v^2 N \left[\frac{5}{256} (\text{tr}(z^A z_A^\dagger))^2 - \frac{1}{32} \text{tr}(z^A z_B^\dagger) \text{tr}(z^B z_A^\dagger) - \frac{1}{32} \text{tr}(z^4 z_4^\dagger) \text{tr}(z^A z_A^\dagger) \right. \\ &\quad \left. + \frac{1}{16} \text{tr}(z^4 z_A^\dagger) \text{tr}(z_4^\dagger z^A) \right] + \frac{v^2}{16} \left[\text{tr} z_A^\dagger (\text{tr} z^B \text{tr}(z^A z_B^\dagger) \right. \\ &\quad \left. - \text{tr} z^4 \text{tr}(z^A z_4^\dagger)) + \text{tr} z^A (\text{tr} z_A^\dagger \text{tr}(z^4 z_4^\dagger) - \text{tr} z_4^\dagger \text{tr}(z^4 z_A^\dagger)) \right] \left. \right\} + \mathcal{O}(v). \end{aligned} \quad (3.54)$$

There appear three types of terms that survive the scaling limit and must be analyzed: First, the cosmological term is blowing up since it is multiplied by $g_M^4 v^6 \sim \bar{g}_M^4 v^2$ but will

nevertheless result in a fixed AdS geometry (also the EH term is $\sim v^2$). Second, there are mass terms $\sim \bar{g}_M^4$ that do not scale. In addition there are terms linear in the scalar fields which are $\sim \bar{g}_M^4 v$. Fortunately this latter kind of term arises also from $R|Z^2|$ and as we will see shortly the two contributions cancel each other.

Collecting the results, the leading terms in v are

$$\begin{aligned} -V_B = & -\frac{g_M^2 v^3 N \lambda}{2\sqrt{2}} f^{abc} X_a^{A+4} X_b^A X_c^8 - \frac{1}{256} g_M^4 v^3 N^3 (v + \sqrt{2} X_0^4)^3 \\ & - \frac{3}{2 \cdot 256} g_M^4 v^2 N^2 (v + \sqrt{2} X_0^4)^2 (N X_0^I X_0^I + X_a^I X_a^I) + \mathcal{O}(v^2). \end{aligned} \quad (3.55)$$

The remaining terms of $\mathcal{O}(v^2)$ are presented in Appendix B. As we have already explained under Eq. (2.28), all terms with f^{abc} also come multiplied by a power of $\lambda = \frac{2\pi}{k}$. This is sensible, since one has from (3.25) that

$$g_{YM} = \frac{2\pi}{k} v \sqrt{2} = \lambda v \sqrt{2}. \quad (3.56)$$

At this point the reader might be alarmed by the fact that the leading contribution from the non-ABJM interaction terms is $\mathcal{O}(v^3)$, while the one for ABJM is $\mathcal{O}(v^2)$. However, one needs to remember that it is the quantity $\bar{g}_M = g_M v$ from (3.45) that we would like to keep fixed in our limit. Additionally, it is necessary to perform the rescaling

$$X_0^I = \frac{\tilde{X}_0^I}{\sqrt{N}}, \quad \psi_0^I = \frac{\tilde{\psi}_0^I}{\sqrt{N}} \quad (3.57)$$

in order to make these of the same order as X_a^I, ψ_a^I . After doing so we obtain that $V_{2-trace, v^2} \sim \mathcal{O}(\lambda) \rightarrow 0$, while $V_{3-trace, v^2} \sim \mathcal{O}(g_M^2) \rightarrow 0$ and indeed the nonzero terms come from the $\mathcal{O}(v^3)$ contributions, giving

$$\begin{aligned} -V_B = & -\frac{\bar{g}_M^2}{4} g_{YM} f^{abc} X_a^{A'} X_b^{A'} X_c^8 - \frac{\bar{g}_M^4 v^2 N}{256} - \frac{3\sqrt{2}}{256} \bar{g}_M^4 v \sqrt{N} \tilde{X}_0^4 - \frac{3}{128} \bar{g}_M^4 (\tilde{X}_0^4)^2 \\ & - \frac{3}{2 \cdot 256} \bar{g}_M^4 (\tilde{X}_0^I \tilde{X}_0^I + X_a^I X_a^I) + \text{subleading}. \end{aligned} \quad (3.58)$$

In the summary of the final results in Section 1.2, these terms appear together with the related terms arising from the expansion of the conformal scalar-curvature term:

$$-\frac{R}{8} |Z|^2 = -\frac{R}{8} \left[N v^2 + v \sqrt{N} \sqrt{2} \tilde{X}_0^4 + \frac{\tilde{X}_0^I \tilde{X}_0^I}{2} + \frac{X_a^I X_a^I}{2} \right]. \quad (3.59)$$

It is important to note here that there are unwanted terms linear in the scalar field \tilde{X}_0^4 in both of (3.58) and (3.59). However, by using the background curvature relation (3.46) the last equation reads

$$-\frac{R_{BG}}{8} |Z|^2 = \frac{3}{256} \bar{g}_M^4 \left[N v^2 + v \sqrt{N} \sqrt{2} \tilde{X}_0^4 + \frac{\tilde{X}_0^I \tilde{X}_0^I}{2} + \frac{X_a^I X_a^I}{2} \right] \quad (3.60)$$

and we find that these linear terms cancel in an expansion around the AdS background. This also affects the values of scalar masses and the bosonic interaction terms as seen from

$$-V_B - \frac{R_{BG}}{8}|Z|^2 = \frac{1}{128}\bar{g}_M^4 v^2 N - \frac{3}{128}\bar{g}_M^4 (\tilde{X}_0^4)^2 - \frac{\bar{g}_M^2}{4} g_{YM} f^{abc} X_a^{A'+4} X_b^{A'} X_c^8 + \text{subleading} . \quad (3.61)$$

As noted already in the previous subsection, the first term in this result is related to the cosmological constant which is proportional to $\bar{g}_M^4 v^2 N$ and thus diverges in the limit used here.¹⁶ The remaining terms are all finite as they should. Moreover, there is no dependence on X_a^4, X_0^8 , exactly as in the pure ABJM part, while X_0^4 is now massive. However, in addition to this pure ABJM-like result there is also no dependence on any of the X_0 scalars except X_0^4 . This indicates that in this topologically gauged case there is an extended Higgs effect at work, where the local SU(4) R-symmetry has also been broken. The coset SU(4)/SU(3) indeed accounts for the missing seven scalars.

In conclusion, we obtain an AdS background geometry that is strongly enforced: Both the Einstein and the cosmological constant terms are multiplied by v^2 , which diverges in the limit used here. Apart from this fact, all the interaction terms between gravity and ABJM are finite.

3.3.2 Fermionic terms

We quote the leading fermionic interaction terms from (B.5), while referring the reader who is interested in the details to Appendix B:

$$\begin{aligned} -V_F = & -\frac{2\sqrt{2}}{\kappa} i A \bar{f}^{\mu A4} \gamma_\mu (\tilde{\psi}_0^A + i\tilde{\psi}_0^{A+4}) - \frac{2\sqrt{2}}{\kappa} \frac{i}{16} \bar{g}_M^2 e A \bar{\chi}_{\mu 4A} \gamma^\mu (\tilde{\psi}_0^A - i\tilde{\psi}_0^{A+4}) + c.c. \\ & + \frac{i\bar{g}_M^2}{16} e \left[3\tilde{\psi}_0^{I''} \tilde{\psi}_0^{I''} + \bar{\psi}_a^{I''} \psi_a^{I''} \right] + \frac{i\bar{g}_M^2}{16} e \left[5(\tilde{\psi}_0^4 \tilde{\psi}_0^4 + \tilde{\psi}_0^8 \tilde{\psi}_0^8) - 3(\bar{\psi}_a^4 \psi_a^4 + \bar{\psi}_a^8 \psi_a^8) \right] \\ & + \frac{4i\bar{g}_M^2}{\kappa^2} \bar{f}_{AB}^{\mu} \chi_\mu^{AB} + \frac{i\bar{g}_M^2}{8\kappa^2} \epsilon^{\mu\nu\rho} \bar{\chi}_{\nu AB} \gamma_\rho \chi_\mu^{AB} + \text{subleading} , \end{aligned} \quad (3.62)$$

with $I'' = \{I \neq 4, 8\}$. These contributions are also all finite. We note that $\tilde{\psi}_0^{I''}, \psi_a^{I''}, \tilde{\psi}_0^{4,8}, \psi_a^{4,8}$ as well as the gravitini have acquired mass terms and that in extracting the leading contributions we have used $A_\mu^- \sim \frac{F_\mu^+}{v^2}$ from (3.34). Also note that the kinetic term for the gravitini has been included in this expression and that, after the breaking to the $\mathbf{3} + \bar{\mathbf{3}}$ of the R-symmetry group SU(3), all six gravitini are still AdS-massless. This fact was incorporated into the supergravity lagrangian quoted in the introductory section, which was claimed to have $\mathcal{N} = 6$ supersymmetry.

3.3.3 The $B_{\mu B}^A, C_\mu$ terms

The gravity sector also contains the fields $B_{\mu B}^A$ and C_μ , which are used to gauge the global symmetries SU(4)_R and U(1) respectively. Although C_μ does not couple to any of the other

¹⁶The second scaling procedure described in the next section gives rise to a finite cosmological constant as well as a freely tunable Newton constant.

gravitational fields including B_{μ}^A [9], it is natural to discuss both of these two chiral CS fields in the gravity context since both groups are obtained when breaking $\text{SO}(8)$.

On top of their kinetic terms, in the presence of a scalar VEV there are also mass terms coming from the ABJM piece, $-e \text{tr}[\tilde{D}_{\mu} Z^A \tilde{D}^{\mu} \bar{Z}_A]$. We have already seen in (3.28) that the fields B_{μ}^A and C_{μ} appear in the covariant derivative as

$$\begin{aligned}\tilde{D}Z^4 &= i[v(B_{\mu 4}^4 + qC_{\mu} + 2A_{0\mu}^-) + \frac{1}{\sqrt{2}}\partial_{\mu}X_0^8] + \dots \\ \tilde{D}Z^{A'} &= [-v\text{Im}(B_{\mu 4}^{A'}) + \frac{1}{\sqrt{2}}\partial_{\mu}X_0^{A'}] + i[v\text{Re}(B_{\mu 4}^{A'}) + \frac{1}{\sqrt{2}}\partial_{\mu}X_0^{A'+4}] + \dots\end{aligned}\quad (3.63)$$

Then, in the absence of a potential for $X_0^{A'}, X_0^{A'+4}, X_0^8$, one can indeed implement the usual Higgs mechanism to have the fields $X_0^{A'}$ and $X_0^{A'+4}$ ‘eaten’ through the redefinition

$$\begin{aligned}\text{Re}(B_{\mu 4}^{A'}) + \frac{1}{\sqrt{2}v}\partial_{\mu}X_0^{A'+4} &\rightarrow \text{Re}(B_{\mu 4}^{A'}) \\ \text{Im}(B_{\mu 4}^{A'}) - \frac{1}{\sqrt{2}v}\partial_{\mu}X_0^{A'} &\rightarrow \text{Im}(B_{\mu 4}^{A'})\end{aligned}\quad (3.64)$$

The gauge fields $B_{\mu 4}^{A'}$ obtain the usual mass terms, while $X_0^{A'}, X_0^{A'+4}$ disappear from the action.

On the other hand, X_0^8 was already ‘eaten’ by $A_{0\mu}^-$ in (3.32), which allowed for the $\text{U}(1)$ part of the nondynamical $\text{U}(N) \times \text{U}(N)$ CS field to turn into the $\text{U}(1)$ part of the dynamical $\text{U}(N)$ Yang-Mills field. However, in that pure gauge theory discussion the effects of B_{μ}^A and C_{μ} were ignored. Accounting for them involves first redefining

$$B_{\mu 4}^4 + qC_{\mu} + 2A_{0\mu}^- = \tilde{A}_{0\mu}^- \quad (3.65)$$

so that

$$\tilde{A}_{0\mu}^- - \frac{1}{\sqrt{2}v}\partial_{\mu}X_0^8 \rightarrow \tilde{A}_{0\mu}^- \quad (3.66)$$

This means that the relevant terms in the action (3.33) become

$$\begin{aligned}&\frac{kN}{2\pi}\varepsilon^{\mu\nu\lambda}A_{\mu 0}^-F_{\nu\lambda}^{+0} - 4Nv^2(\tilde{A}_{\mu 0}^- + \frac{1}{\sqrt{2}v}\partial_{\mu}X_0^8)^2 \\ \rightarrow &\frac{kN}{4\pi}\varepsilon^{\mu\nu\lambda}\tilde{A}_{\mu 0}^-F_{\nu\lambda}^{+0} - 4Nv^2\tilde{A}_{\mu 0}^-\tilde{A}_0^{-\mu} - \frac{kN}{4\pi}\varepsilon^{\mu\nu\lambda}(qC_{\mu} + B_{\mu 4}^4)F_{\nu\lambda}^{+0}\end{aligned}\quad (3.67)$$

As was the case before the $\tilde{A}_{0\mu}^-$ terms give the $\text{U}(1)$ part of the Yang-Mills action, while the extra one is

$$-\frac{kN}{4\pi}\varepsilon^{\mu\nu\lambda}(qC_{\mu} + B_{\mu 4}^4)F_{\nu\lambda}^{+0} = -\frac{2\sqrt{N}}{g_{YM}\kappa}\varepsilon^{\mu\nu\lambda}(qC_{\mu} + B_{\mu 4}^4)F_{\nu\lambda}^{+0} \quad (3.68)$$

Rewriting everything in terms of κ and μ , we finally get the B, C action

$$\begin{aligned}&\frac{1}{\kappa^2\mu^2}\int d^3x\left[-\frac{\mu}{2}\varepsilon^{\mu\nu\rho}C_{\mu}\partial_{\nu}C_{\rho} - 2\mu\varepsilon^{\mu\nu\rho}\text{Tr}_A(B_{\mu}\partial_{\nu}B_{\rho} + \frac{2}{3}B_{\mu}B_{\nu}B_{\rho})\right. \\ &\quad \left.- 8\mu^2|B_{\mu 4}^{A'}|^2 - \frac{4\sqrt{N}\mu\kappa^2\mu^2}{g_{YM}}\varepsilon^{\mu\nu\lambda}(qC_{\mu} + B_{\mu 4}^4)F_{\nu\lambda}^{+0}\right],\end{aligned}\quad (3.69)$$

where the SU(4) (anti-)hermitian traceless field $B_{\mu B}^A$ has been split into a massless SU(3) (anti-)hermitian traceless part $B_{\mu B'}^{A'}$, the massless 3×3 trace $iB_{0\mu} \equiv \sum_{A'} B_{\mu A'}^{A'} = -B_{\mu 4}^4$, and the massive complex vector $B_{\mu 4}^{A'}$ (which ate the $X_0^{A'}, X_0^{A'+4}$ fields). Note that $B_{\mu A'}^4$ is related to $B_{\mu 4}^{A'}$ by the (anti-)hermiticity condition.

Under this decomposition the CS term for the SU(4) field $B_{\mu B}^A$ splits into SU(3) components as

$$-\frac{2}{\mu}\varepsilon^{\mu\nu\rho}\text{Tr}_A(B_\mu\partial_\nu B_\rho + \frac{2}{3}B_\mu B_\nu B_\rho) = -\frac{2}{\mu}\varepsilon^{\mu\nu\rho}\text{Tr}_{A'}(B_\mu\partial_\nu B_\rho + \frac{2}{3}B_\mu B_\nu B_\rho) - \frac{4}{\mu}\varepsilon^{\mu\nu\rho}B_{\mu 4}^{A'}\check{D}_\nu B_{\rho A'}^4 + \frac{2}{\mu}\varepsilon^{\mu\nu\rho}B_{0\mu}\partial_\nu B_{0\rho}, \quad (3.70)$$

where

$$\check{D}_\nu B_{\rho A'}^4 \equiv \partial_\nu B_{\rho A'}^4 + iB_{0\nu}B_{\rho A'}^4 - \frac{2}{3}B_{\nu A'}^{B'}B_{\rho B'}^4. \quad (3.71)$$

It is evident that the SU(4) R-symmetry has been broken to SU(3) \times U(1). At the same time one combination of the abelian vector fields becomes massive. This field is still denoted A_μ^+ although it is a sum of several fields. Hence there is just one scalar degree of freedom being absorbed in the higgsing of the abelian sector. Note that despite the appearance of the coupling term between F^+ and $(qC + B)$ there is no further breaking of the U(1) R-symmetry. The counting of degrees of freedom is then easily understood in terms of the $6 + N^2$ dimensional coset $\text{SU}(4) \times \text{U}(1) \times \text{U}(N) \times \text{U}(N)/\text{SU}(3) \times \text{U}(1) \times \text{U}(1) \times \text{U}(N)$.

The above discussion relied on the fact that massive CS-theories in three dimensions have been analyzed in detail and seen to have one degree of freedom. Thus the action for the $B_{\mu 4}^{A'}$ field is the ‘self-dual action in odd dimensions’ of [35]. For an abelian field A_μ one could use the transformation of [36], which allows to express an abelian self-dual field as a Maxwell-CS (topologically massive) one, *i.e.*

$$\frac{1}{\kappa^2\mu^2} \int d^3x \left[-\frac{1}{4}F_{\mu\nu}^2 - \frac{\mu}{2}\varepsilon^{\mu\nu\rho}A'_\mu\partial_\nu A'_\rho \right], \quad (3.72)$$

where $F_{\mu\nu} = \partial_{[\mu}A'_{\nu]}$.

For nonabelian, coupled fields like our $B_{\mu 4}^{A'}$ this transformation cannot be performed [37].¹⁷ However, there is nothing unusual with having self-dual actions in odd dimensions in the context of supergravity. In fact, such an action appears *e.g.* in 7 dimensional gauged supergravity and it was shown in [38, 39] that it can arise by dimensional reduction of the action of 11 dimensional supergravity.¹⁸

¹⁷The action for $B_{\mu 4}^{A'}$ contains the terms $B_{\mu 4}^{A'}B_{\nu B'}^4B_{\rho A'}^{B'}$.

¹⁸In odd dimensions the usual Klein-Gordon kinetic operator splits into two self-dual parts of opposite relative sign

$$\square - m^2 \sim (\epsilon\partial + m)(\epsilon\partial - m), \quad (3.73)$$

which means that a field with the Klein-Gordon equation of motion $(\square - m^2)A = 0$ can be split, by further adding an auxiliary field B with action B^2 , into two equations of motion of self-dual type

$$(\epsilon\partial + m)P = 0 \quad \text{and} \quad (\epsilon\partial - m)Q = 0. \quad (3.74)$$

These equations are in turn coming from an action of the type $\sim [\epsilon P\partial P + mP^2 - \epsilon Q\partial Q + mQ^2]$. In this sense, the self-dual kinetic operator is the ‘square root’ of the Klein-Gordon one.

3.4 Supersymmetry rules

We finally turn our attention to the effect of the higgsing on the supersymmetry transformation rules. Since both the $U(N) \times U(N)$ gauge fields A_μ and the $B_\mu^{A'}$ fields have ‘eaten’ scalars to become dynamical, it is important to clarify how the transformations rules organize the physical states into $\mathcal{N} = 6$ supermultiplets. The answer to this question will, however, require additional information about the exact form of the special superconformal transformations leaving the unhiggsed theory invariant.

As we will see below there is an interplay between the supersymmetry and special conformal supersymmetry transformations that needs to be understood in order to determine the form of the supersymmetry transformations that are left unbroken after higgsing of the theory. The introduction of a VEV with non-zero mass dimension breaks the conformal and dilation dependent symmetries, in particular the special superconformal symmetry. However, this does not imply that the supersymmetry that remains after higgsing coincides exactly with that called supersymmetry in the conformal theory. Instead, in the analysis below we find several facts that point towards one and the same solution to the question of how to define supersymmetry after higgsing. This solution involves forming a particular linear combination of the original supersymmetry and the special conformal supersymmetry transformations.

3.4.1 The effect of the VEV on the supersymmetry of the original fields

In this subsection we will first derive the rules in terms of the original fields, *i.e.* before the ‘eating’ has been implemented. In fact, this latter step would mean identifying the final supermultiplets and that we will not be able to do in this work since we lack a full control of the special superconformal transformations in the ABJM theory coupled to conformal gravity, *i.e.* of the topologically gauged theory. The special superconformal transformations are easily constructed in both the pure $\mathcal{N} = 6$ superconformal gravity theory, as demonstrated in [10, 9], as well as in the flat space ABJM theory [40]. However, the validity of these expressions have not yet been verified in the coupled theory.

By replacing the Higgs ansatz in the supersymmetry rules (2.37)-(2.40) we obtain

$$\begin{aligned}
\delta e_\mu^\alpha &= i\bar{\epsilon}_{gAB}\gamma^\alpha\chi_\mu^{AB} \\
\delta\chi_\mu^{AB} &= \tilde{D}_\mu\epsilon_g^{AB} \\
\delta B_\mu^A &= \frac{i}{e}(\bar{f}^{\nu AC}\gamma_\mu\gamma_\nu\epsilon_{gBC} - \bar{f}_{BC}^\nu\gamma_\mu\gamma_\nu\epsilon_g^{AC}) \\
&\quad + \frac{i}{4\kappa}\left[\delta^{A4}\bar{\epsilon}_{BD}\gamma_\mu(\tilde{\psi}_0^D - i\tilde{\psi}_0^{D+4}) + \bar{\epsilon}_{B4}\gamma_\mu(\tilde{\psi}_0^A - i\tilde{\psi}_0^{A+4})\right. \\
&\quad \left.- \delta_{B4}\bar{\epsilon}^{AD}\gamma_\mu(\tilde{\psi}_0^D + i\tilde{\psi}_0^{D+4}) - \bar{\epsilon}^{A4}\gamma_\mu(\tilde{\psi}_0^B + i\tilde{\psi}_0^{B+4})\right] \\
&\quad - \frac{4i}{\kappa^2}(\bar{\epsilon}_g^{AC}\chi_{\mu 4C}\delta_{B4} - \bar{\epsilon}_{gBC}\chi_\mu^{4C}\delta^{A4}) + \frac{i}{\kappa^2}(\bar{\epsilon}_g^{AD}\chi_{\mu BD} - \bar{\epsilon}_{gBD}\chi_\mu^{AD}) \\
\delta C_\mu &= -vq(\bar{\epsilon}_{A4}\gamma_\mu\text{tr}(\Psi^{\dagger A}) - \bar{\epsilon}^{A4}\gamma_\mu\text{tr}(\Psi_A)) - \frac{16}{\kappa^2}q(\bar{\epsilon}_g^{AD}\chi_{\mu 4D} - \bar{\epsilon}_{g4D}\chi_\mu^{AD}) \\
&= -\frac{2q}{\kappa}\left[\bar{\epsilon}_{A4}\gamma_\mu(\tilde{\psi}_0^A - i\tilde{\psi}_0^{A+4}) - \bar{\epsilon}^{A4}\gamma_\mu(\tilde{\psi}_0^A + i\tilde{\psi}_0^{A+4})\right]
\end{aligned}$$

$$\begin{aligned}
& -\frac{16}{\kappa^2} q(\bar{\epsilon}_g^{4D} \chi_{\mu 4D} - \bar{\epsilon}_{g4D} \chi_{\mu}^{4D}) \\
\delta(\tilde{X}_0^A + i\tilde{X}_0^{A+4}) &= i\bar{\epsilon}^{AB}(\tilde{\psi}_0^B + i\tilde{\psi}_0^{B+4}) \\
\delta(X_a^A + iX_a^{A+4}) &= i\bar{\epsilon}^{AB}(\psi_a^B + i\psi_a^{B+4}) .
\end{aligned} \tag{3.75}$$

For Ψ_B of (2.41) and the combined $A_\mu^{L/R}$ of (2.45)-(2.46) we only write the new terms (gravitational interaction terms)

$$\begin{aligned}
\delta\Psi_B - \delta_0\Psi_B &= \frac{\bar{g}_M^2 v}{16} \epsilon_{4B} + \frac{\bar{g}_M^2}{4\sqrt{2}} \delta_{B4} \left[\frac{(\tilde{X}_0^A + i\tilde{X}_0^{A+4})}{\sqrt{N}} (\epsilon_{A4} + \epsilon_{4A}) + (iX_a^A - X_a^{A+4}) T^a \epsilon_{4A} \right] \\
&+ \frac{\bar{g}_M^2}{16\sqrt{2}} \left[\frac{(\tilde{X}_0^A + i\tilde{X}_0^{A+4})}{\sqrt{N}} \epsilon_{AB} + \frac{2\tilde{X}_0^4}{\sqrt{N}} \epsilon_{AB} + (iX_a^A - X_a^{A+4}) \epsilon_{AB} T^a \right] \\
\delta A_\mu^L - \delta_0 A_\mu^L &= \delta A_\mu^R - \delta_0 A_\mu^R = \frac{8i}{\kappa} \left[-\frac{\tilde{X}_0^{A+4}}{\sqrt{N}} + X_a^A T^a \right] (\bar{\epsilon}_g^{4D} \chi_{\mu AD} - \bar{\epsilon}_{gAD} \chi_{\mu}^{4D}) ,
\end{aligned} \tag{3.76}$$

noting that in the first equation the interaction term $iA\gamma^\mu \epsilon_{AB} \bar{\chi}^{BD} \Psi_D$ vanishes under rescaling and higgsing. Expanding Ψ_B we get

$$\begin{aligned}
\delta(\tilde{\psi}_0^B + i\tilde{\psi}_0^{B+4}) - \delta_0(\tilde{\psi}_0^B + i\tilde{\psi}_0^{B+4}) &= \frac{\sqrt{2N}\bar{g}_M^2}{16} \left(v + \frac{\sqrt{2}\tilde{X}_0^4}{\sqrt{N}} \right) \epsilon_{4B} + \frac{\bar{g}_M^2}{16} (\tilde{X}_0^A + i\tilde{X}_0^{A+4}) \epsilon_{AB} \\
\delta(i\psi_a^B - \psi_a^{B+4}) - \delta_0(i\psi_a^B - \psi_a^{B+4}) &= \frac{\bar{g}_M^2}{16} (iX_a^A - X_a^{A+4}) (\epsilon_{AB} + 4\epsilon_{4A} \delta_{B4}) .
\end{aligned} \tag{3.77}$$

In the above, δ_0 denote the set of supersymmetry transformations for the pure ABJM theory. Pre-higgsing these were given by

$$\begin{aligned}
\delta_0\Psi_A &= -\gamma_\mu \epsilon_{AB} \tilde{D}_\mu Z^B + \lambda(-\epsilon_{AB}(Z^C Z_C^\dagger Z^B - Z^B Z_C^\dagger Z^C) + 2\epsilon_{CD} Z^C Z_A^\dagger Z^D) \\
\delta_0 A_\mu^L &= \lambda(\bar{\epsilon}_{AB} \gamma_\mu Z^B \Psi^{A\dagger} - \bar{\epsilon}^{AB} \gamma_\mu \Psi_A Z_B^\dagger) \\
\delta_0 A_\mu^R &= \lambda(\bar{\epsilon}_{AB} \gamma_\mu \Psi^{A\dagger} Z^B - \bar{\epsilon}^{AB} \gamma_\mu Z_B^\dagger \Psi_A) .
\end{aligned}$$

Post-higgsing, the v contribution cancels between the two terms in the gauge field transformations, hence

$$\begin{aligned}
\delta_0 A_\mu^L &= \lambda(\bar{\epsilon}_{AB} \gamma_\mu z^B \Psi^{A\dagger} - \bar{\epsilon}^{AB} \gamma_\mu \Psi_A z_B^\dagger) \\
\delta_0 A_\mu^R &= \lambda(\bar{\epsilon}_{AB} \gamma_\mu \Psi^{A\dagger} z^B - \bar{\epsilon}^{AB} \gamma_\mu z_B^\dagger \Psi_A) .
\end{aligned} \tag{3.78}$$

For the fermions we have

$$\begin{aligned}
\delta_0\Psi_A &= iv\gamma_\mu \epsilon_{BA} [2\delta^{B4} A_\mu^- + (B_{\mu 4}^B + qC_\mu \delta^{B4})] - \gamma_\mu \epsilon_{AB} \tilde{D}_\mu z^B \\
&+ \sqrt{2} g_{YM} (\delta_{A4} \epsilon_{CD} z^C z^D + \epsilon_{C4} [z^C, z_A^\dagger]) - \frac{g_{YM}}{\sqrt{2}} (\epsilon_{A4} [z^C, z_C^\dagger] + \epsilon_{AB} [z^4 + z_4^\dagger, z^B]) \\
&= \left[\frac{2\sqrt{2}i}{\kappa\sqrt{N}} (B_{\mu 4}^B - B_{\mu 4}^4 \delta^{B4}) + \frac{i}{2\sqrt{2}g_{YM}} \varepsilon_{\mu\nu\rho} F^{+\nu\rho} \delta^{B4} \right] \gamma^\mu \epsilon_{BA} \\
&- \frac{1}{\sqrt{2}} \gamma_\mu \epsilon_{AB'} \partial_\mu \left[\frac{1}{\sqrt{N}} (X_0^{B'} + iX_0^{B'+4}) + (iX_a^{B'} - X_a^{B'+4}) T^a \right] \\
&- \frac{1}{\sqrt{2}} \gamma_\mu \epsilon_{A4} \partial_\mu \left[\frac{1}{\sqrt{N}} X_0^4 - X_a^8 T^a \right] \\
&+ \sqrt{2} g_{YM} \left[\frac{1}{4} \delta_{A4} \epsilon_{CD} (iX_a^C - X_a^{C+4}) (iX_b^D - X_b^{D+4}) \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \epsilon_{C4} (iX_a^C - X_a^{C+4}) (-iX_b^A - X_b^{A+4})] f^{ab}{}_c T^c \\
& - \frac{g_{YM}}{\sqrt{2}} (\epsilon_{A4} X_a^C X_b^{C+4} + i\epsilon_{AB} (iX_a^B - X_a^{B+4}) X_b^8 f^{ab}{}_c T^c).
\end{aligned} \tag{3.79}$$

This last set of higgsed pure ABJM transformations should be compared to the ones for three-dimensional SYM in the low-energy limit of N D2-branes in flat space.

3.4.2 Supersymmetry for redefined higgsed fields

We can now use (3.32), (3.64) and (3.66) to obtain the transformations for the higgsed fields after the ‘eating’ redefinition. Since the $A_{\mu a}^-$ and $A_{\mu 0}^-$ fields get integrated out, the bosonic on-shell degrees of freedom are contained in $X_a^{I'}$, X_0^4 , $A_{a\mu}^+$, $A_{0\mu}^+$ and $B_{\mu 4}^{A'}$. The above redefinitions affect the supersymmetry rules through the change

$$\delta B_{\mu 4}^{A'} - \frac{1}{\sqrt{2}v} \partial_\mu \delta (X_0^{A'} + iX_0^{A'+4}) = \delta \hat{B}_{\mu 4}^{A'} \tag{3.80}$$

and one also needs to drop the $X_0^{A'}, X_0^{A'+4}, X_0^8$ transformation rules altogether.¹⁹ The transformations for the bosonic fields then read

$$\begin{aligned}
\delta e_\mu^\alpha &= i\bar{\epsilon}_{gAB} \gamma^\alpha \chi_\mu^{AB} \\
\delta \chi_\mu^{AB} &= \tilde{D}_\mu \epsilon_g^{AB} \\
\delta B_{\mu B'}^{A'} &= \frac{i}{e} (\bar{f}^{\nu A'C} \gamma_\mu \gamma_\nu \epsilon_{gB'C} - \bar{f}_{B'C}^\nu \gamma_\mu \gamma_\nu \epsilon_g^{A'C}) + \frac{i}{\kappa^2} (\bar{\epsilon}_g^{A'D} \chi_{\mu B'D} - \bar{\epsilon}_{gB'D} \chi_\mu^{A'D}) \\
\delta B_{\mu 4}^{A'} &= \frac{i}{e} (\bar{f}^{\nu A'C} \gamma_\mu \gamma_\nu \epsilon_{g4C} - \bar{f}_{4C}^\nu \gamma_\mu \gamma_\nu \epsilon_g^{A'C}) + \frac{\kappa\sqrt{N}}{4} \partial_\mu [\bar{\epsilon}^{AB} (\tilde{\psi}_0^B + i\tilde{\psi}_0^{B+4})] \\
&\quad + \frac{i}{4\kappa} \left[-\bar{\epsilon}^{A'D} \gamma_\mu (\tilde{\psi}_0^D + i\tilde{\psi}_0^{D+4}) - \bar{\epsilon}^{A'4} \gamma_\mu (\tilde{\psi}_0^4 + i\tilde{\psi}_0^8) \right] \\
&\quad - \frac{4i}{\kappa^2} \bar{\epsilon}_g^{A'C} \chi_{\mu 4C} + \frac{i}{\kappa^2} (\bar{\epsilon}_g^{A'D} \chi_{\mu 4D} - \bar{\epsilon}_{g4D} \chi_\mu^{A'D}) \\
\delta B_{0\mu} \equiv i\delta B_{\mu 4}^4 &= -\frac{1}{e} (\bar{f}^{\nu 4C} \gamma_\mu \gamma_\nu \epsilon_{g4C} - \bar{f}_{4C}^\nu \gamma_\mu \gamma_\nu \epsilon_g^{4C}) \\
&\quad - \frac{4}{\kappa} \left[\bar{\epsilon}_{4D} \gamma_\mu (\tilde{\psi}_0^D - i\tilde{\psi}_0^{D+4}) - \bar{\epsilon}^{4D} \gamma_\mu (\tilde{\psi}_0^D + i\tilde{\psi}_0^{D+4}) \right] \\
&\quad - \frac{1}{\kappa^2} (\bar{\epsilon}_g^{4D} \chi_{\mu 4D} - \bar{\epsilon}_{g4D} \chi_\mu^{4D}) \\
\delta C_\mu &= -\frac{2q}{\kappa} \left[\bar{\epsilon}_{A4} \gamma_\mu (\tilde{\psi}_0^A - i\tilde{\psi}_0^{A+4}) - \bar{\epsilon}^{A4} \gamma_\mu (\tilde{\psi}_0^A + i\tilde{\psi}_0^{A+4}) \right] \\
&\quad - \frac{16}{\kappa^2} q (\bar{\epsilon}_g^{4D} \chi_{\mu 4D} - \bar{\epsilon}_{g4D} \chi_\mu^{4D}) \\
\delta \tilde{X}_0^4 &= i\bar{\epsilon}^{4B'} \tilde{\psi}_0^{B'} \\
\delta X_a^{A'} &= i\bar{\epsilon}^{A'B} \psi_a^B \\
\delta X_a^{A+4} &= i\bar{\epsilon}^{AB} \psi_a^{B+4} \\
\delta A_{0\mu}^+ &= -\frac{8}{\kappa\sqrt{N}} \tilde{X}_0^{A+4} (\bar{\epsilon}_g^{4D} \chi_{\mu AD} - \bar{\epsilon}_{gAD} \chi_\mu^{4D}) \\
&\quad + \frac{\lambda}{2} (\bar{\epsilon}_{AB} \gamma_\mu \frac{1}{N} \text{tr}[\{z^B, \Psi^{A\dagger}\}] - \bar{\epsilon}^{AB} \gamma_\mu \frac{1}{N} \text{tr}[\{\Psi_A, z_B^\dagger\}])
\end{aligned}$$

¹⁹For notational simplicity we will drop the hat on $B_{\mu 4}^{A'}$ after the redefinition, with the hope that this will not cause any confusion.

$$\begin{aligned}\delta A_{a\mu}^+ &= \frac{8}{\kappa} X_a^A (\bar{\epsilon}_g^{AD} \chi_{\mu AD} - \bar{\epsilon}_{gAD} \chi_{\mu}^{AD}) \\ &\quad + \frac{\lambda}{2} (\bar{\epsilon}_{AB} \gamma_{\mu} \{z^B, \Psi^{A\dagger}\}_a - \bar{\epsilon}^{AB} \gamma_{\mu} \{\Psi_A, z_B^{\dagger}\}_a) .\end{aligned}\tag{3.81}$$

The above set of transformations, along with the ones for the fermionic fields presented in the previous subsection,²⁰ should leave the final action invariant. The situation should be completely analogous to the case of pure ABJM in the limit of large v , where the supersymmetry transformations post-higgsing can be seen to exactly reduce to the ones for three-dimensional SYM.

However, the more intriguing properties of the above transformation rules are found by looking at the transformation rules for the spinors. The gravitini transformation differs from the one expected in a theory with an AdS background in that the covariant derivative in $\delta \chi_{\mu}^{AB} = \tilde{D}_{\mu} \epsilon_g^{AB}$ does not contain a term proportional to a three-dimensional gamma matrix. Such a term appears however in the special superconformal transformations of the pure $\mathcal{N} = 6$ supergravity theory [10, 9] and by combining this with the original supersymmetry transformations given above the correct form of the variation of the gravitini could be obtained. By correct we mean that the resulting Killing spinor equation should have six solutions corresponding to the number of expected supersymmetries in the AdS background.

There is another fact pointing in the same direction, namely the variation of the spinor field from the ABJM theory. As seen above, the ordinary supersymmetry in the conformal theory has picked up a constant shift after higgsing. Such a term would normally indicate that supersymmetry is broken and that the fermion in question is a Goldstone fermion. However, in our case we can eliminate this constant shift by adding a special superconformal transformation in the same way as described in the previous paragraph.

The conclusion is that the supersymmetry that survives the higgsing of the superconformal theory is a linear combination of the supersymmetry Q and the special conformal supersymmetry S in the superconformal symmetry algebra of the unhiggsed theory, *i.e.* the original topologically gauged ABJM theory. To make this explicit, the special superconformal transformations identified in [10, 9] read

$$\delta \chi_{\mu AB} = \gamma_{\mu} \eta_{AB}\tag{3.82}$$

and the corresponding transformation for the ABJM spinor field is [40]

$$\delta \Psi_A = Z^B \eta_{AB} ,\tag{3.83}$$

which in fact is the unique expression for this transformation. The arguments presented above translate into the following result:

$$Q(\epsilon_{AB}^{\text{AdS}}) = Q(\epsilon_{AB}^{\text{conf}}) + S(\eta_{AB}^{\text{conf}} = \frac{\bar{g}_M^2}{16} \epsilon_{AB}^{\text{conf}}) .\tag{3.84}$$

²⁰Note that the term $\frac{\bar{g}_M^2}{16} (\tilde{X}_0^A + i\tilde{X}_0^{A+4}) \epsilon_{AB}$ in the fermion variation (3.77) is the one needed to cancel the variation of the background term $-\frac{1}{8} R_{BG} |Z|^2$. Also, the $X_0^{A'}$, $X_0^{A'+4}$ terms in (3.79) get absorbed in $B_{\mu}^{A'}$ by the ‘eating’ redefinition.

Recalling now that $\frac{\bar{g}_M^4}{64} = \frac{1}{\ell^2}$ we conclude that the supersymmetry transformation of the gravitini in AdS becomes

$$\delta\chi_{\mu AB} = (\tilde{D}_\mu + \frac{1}{2\ell}\gamma_\mu)\epsilon_{AB}, \quad (3.85)$$

where we have chosen the positive root of \bar{g}_M^4 . This choice will be further justified in Section 4.1. With this variation of the gravitini we get a Killing spinor equation that has six solutions, that is, the theory has six supersymmetries as expected (see *e.g.* [41]). This result also fits well with the corresponding formula in $\mathcal{N} = 1$ chiral supergravity [20].

We end this section with an observation on the multiplet structure. Before performing the higgsing, we started with two interacting $\mathcal{N} = 6$ supermultiplets: the ABJM one, with 8 bosonic and 8 fermionic on-shell degrees of freedom, and the topological conformal supergravity one, with no on-shell degrees of freedom. After the higgsing, due to the R-symmetry fields of the topological multiplet eating scalars from the abelian part of the ABJM multiplet, we have a single entangled $\mathcal{N} = 6$ supermultiplet with the same on-shell degrees of freedom as the original ABJM multiplet.²¹ We are not aware of such examples being discussed in the literature, so we will not try to fit this final multiplet into a general analysis. The fact that the various fields in this multiplet have different masses is typical of AdS backgrounds [42, 41].

4. Interpreting the supergravity action

Let us summarize our findings thus far: After higgsing the topologically gauged ABJM theory we obtained an $\mathcal{N} = 6$ action for three-dimensional SYM/CS theory coupled to supergravity. The gravity sector is interesting since it consists of an EH kinetic term plus a cosmological constant on top of the conformal gravity part that we started off with. As we will discuss shortly this is a supersymmetric extension of the chiral gravity of [11] coupled to a SYM/CS theory.

We begin with a few more comments on the effects of the nontrivial gravitational background on the physics. In general dimension d , a term in the action of the type

$$\frac{1}{16\pi G} \int (R - 2\Lambda) \quad (4.1)$$

leads to Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\Lambda g_{\mu\nu}. \quad (4.2)$$

Tracing the above we get

$$R = \frac{2d}{d-2}\Lambda, \quad (4.3)$$

which gives rise to the relation

$$\Lambda = -\frac{(d-1)(d-2)}{2\ell^2} \quad (4.4)$$

²¹In the language of the simple WZ analogy, after the redefinition one of the scalar degrees of freedom in the WZ multiplet gets turned into a degree of freedom for the gauge field in the vector multiplet.

between the AdS_d radius ℓ defined by $R_{\mu\nu}{}^{\rho\sigma} = -\frac{1}{\ell^2}(\delta_\mu^\rho\delta_\nu^\sigma - \delta_\mu^\sigma\delta_\nu^\rho)$ and the cosmological constant.

Because of the presence of the gravitational CS part in the three-dimensional theory studied here, the Einstein equations will in general also involve the Cotton tensor $\mathcal{C}_{\mu\nu}$, as in (3.39), which however vanishes on the conformally flat AdS_3 background. Moreover, since then $R = 6\Lambda$ one has from (3.41) that (recalling the definition $\bar{g}_M^2 = g_M^2 v^2 N$)

$$\Lambda = -\frac{1}{\ell^2} = -\frac{\bar{g}_M^4}{64} = \text{fixed} , \quad (4.5)$$

which means we are in a finite curvature AdS space and confirms our result (3.43).

In AdS_d space the Breitenlohner-Freedman (BF) bound for scalars allows to a certain extent for a negative mass squared. The bound relates the mass to the curvature through

$$m^2 \geq -\frac{1}{\ell^2} \frac{(d-1)^2}{4} , \quad (4.6)$$

which in our case of $d = 3$ gives

$$m^2 \geq -\left(\frac{\bar{g}_M^2}{8}\right)^2 \equiv -\mu^2 \quad (4.7)$$

and we would like to note the condition $\mu\ell = 1$ found in the previous section. Hence the BF bound is always respected by the scalars in the higgsed theory, since they are either massless or massive (\tilde{X}_0^4) but never ‘tachyonic’, that is, never have a negative mass squared.

4.1 Comparison with the chiral gravity and chiral $\mathcal{N}=1$ supergravity literature

By adding to the lagrangian (3.44) the necessary fermionic terms coming from the third line of (3.62), we recover the full pure gravity sector of the $\mathcal{N} = 6$ action, which was given explicitly in (1.2). Let us however restrict ourselves to the graviton plus single gravitino terms and put the rest of the fields to their VEVs. This leads to an $\mathcal{N} = 1$ supergravity which can be expressed as

$$\begin{aligned} \mathcal{L} = & \left[-e \frac{v^2 N}{8} (R - 2\Lambda) + \frac{v^2 N}{8} \frac{1}{2\mu} \text{Tr}_\alpha (\tilde{\omega} \wedge d\tilde{\omega} + \frac{2}{3} \tilde{\omega} \wedge \tilde{\omega} \wedge \tilde{\omega}) \right] \\ & - 2i \frac{v^2 N}{8} \left[\frac{e}{2\mu} \tilde{D}_\mu \bar{\chi}_\nu \gamma^{\rho\sigma} \gamma^{\mu\nu} \tilde{D}_\rho \chi_\sigma - \epsilon^{\mu\nu\rho} \bar{\chi}_\mu \left(\tilde{D}_\nu + \frac{1}{2\ell} \gamma_\nu \right) \chi_\rho \right] . \end{aligned} \quad (4.8)$$

A similar action was derived by Becker *et al.* in [20] as a (1,0) supersymmetric extension of the chiral pure gravity case considered in [11]. The lagrangian of [20] reads in our conventions²²

$$\begin{aligned} \mathcal{L}' = & \frac{1}{16\pi G} \left[e(R - 2\Lambda) + \frac{1}{2\mu} \text{Tr}_\alpha (\tilde{\omega} \wedge d\tilde{\omega} + \frac{2}{3} \tilde{\omega} \wedge \tilde{\omega} \wedge \tilde{\omega}) \right] \\ & - \frac{i}{16\pi G} \left[\frac{e}{2\mu} \tilde{D}_\mu \bar{\chi}_\nu \gamma^{\rho\sigma} \gamma^{\mu\nu} \tilde{D}_\rho \chi_\sigma + \epsilon^{\mu\nu\rho} \bar{\chi}_\mu \left(\tilde{D}_\nu - \frac{1}{2\ell} \gamma_\nu \right) \chi_\rho \right] . \end{aligned} \quad (4.9)$$

²²Reference [20] uses a different conventions for the fermions: $\bar{\psi} = \psi^\dagger \gamma_0 = -\psi^\dagger \gamma^0$ and for $\epsilon^{012} = -1$, while our conventions are $\bar{\psi} = \psi^\dagger \gamma^0$ and $\epsilon^{012} = +1$.

One immediately sees that by rescaling the gravitino by a factor of $1/\sqrt{2}$, and identifying

$$\frac{1}{\kappa^2} \equiv \frac{1}{16\pi G} = \frac{v^2 N}{8} \quad (4.10)$$

the two actions are identical up to some signs. The relative sign between the covariant derivative and the gamma matrix differ in the two cases but this is just a matter of whether we have $(6, 0)$ or $(0, 6)$ supersymmetry and is not important here.

Another sign difference with important physical implications is, however, expected in view of an observation made already in [9] related to the sign of R . This sign was taken to be $+$ in [11, 20], which is opposite to the usual conventions in the literature on TMG and the one appearing in the theory discussed here. The purpose of choosing the $+$ sign was to have positive energy BTZ black holes but negative energy massive gravitons. However, [11] made probable that when the theory sits at the chiral point $\mu\ell = 1$ the massive gravitons become zero energy states and one is led to a more sensible theory with a conjectured chiral-CFT dual. This point of view generated some controversy in the literature which is summarized in [14], where the reader can find an extensive list of references. In fact, it is still a matter of debate whether the boundary dual to the above theory is a chiral or logarithmic CFT, depending on the subtle choice of gravitational boundary conditions discussed also in [15, 43, 21].

Our theory on the other hand is a TMG theory which appears at the chiral point $\mu\ell = 1$ as an automatic consequence of higgsing the topologically gauged ABJM theories. This suggests a reversal of the signs of the energies for the gravity states and thus leads to zero energy gravitons together with negative energy (unstable) black holes, as is customary from the three-dimensional TMG literature. These black holes are potentially unpleasant and attempts have been made to argue that they can never be formed in physical processes, see for instance [16]. Chirality might in our case, as in the pure chiral gravity case, eliminate the propagating gravitational bulk modes although this analysis has not been carried out for the $\mathcal{N} = 6$ supersymmetric situation found here. In any case, our construction provides an $\mathcal{N} = 6$ generalization of previous models [11, 20] and a setup for investigating these issues from other angles.

A final question concerns the supersymmetry transformation rules for the gravitini and the form of the AdS covariant derivative. Recall that we concluded in the previous section that after the VEV is introduced we get

$$\delta\chi_\mu^{AB} = \left(\tilde{D}_\mu + \frac{1}{2\ell}\gamma_\mu\right)\epsilon_g^{AB}, \quad (4.11)$$

appropriate for a gravitational theory with a (negative) cosmological constant. However, the relative sign between the covariant derivative and the gamma matrix was determined by the analysis of the transformation rules in the previous section and must be consistent with the terms in the action containing the same kind of AdS combination of a derivative and gamma matrix.

First, of course, the sign is given by the gravitini kinetic terms presented in the beginning of this subsection and we find also in this case a relative $+$ sign between the covariant

derivative and the gamma matrix. There are, however, other terms containing generalized AdS derivatives. By looking at our full $\mathcal{N} = 6$ lagrangian, we see that there exists a mixing term coming from the first line of (3.62) and involving the spin- $\frac{1}{2}$ fermions:

$$\left[\bar{f}^{\mu A 4} - e \frac{\bar{g}_M^2}{16} \bar{\chi}^{\mu A 4} \right] \gamma_\mu (\psi_0^A + i \psi_0^{A+4}) , \quad (4.12)$$

which can be rewritten by spinor flipping and seen to contain

$$\gamma^\mu \left[f_\mu^{AB} - \frac{e}{2\ell} \chi_\mu^{AB} \right] = \frac{e}{2} \gamma^{\nu\mu} \left(\tilde{D}_\nu + \frac{1}{2\ell} \gamma_\nu \right) \chi_\mu^{AB} , \quad (4.13)$$

thus confirming that the relative sign is $+$. Having firm evidence of the form of the supersymmetry transformation rules we conclude that this AdS theory has six chiral supersymmetries²³ which we might denote as $(0,6)$. The above terms in the action are included in the final form of the higgsed action presented in equations (1.1) to (1.6).

4.2 Gravity interaction with matter

We next try to understand how gravity interacts with matter. The theory has two gravitational-type coupling parameters: The usual Newton coupling κ from (4.10), which goes to zero when the limits are taken but is kept non-zero in most of our analysis, and the finite $\bar{g}_M^2 \sim \mu$, which multiplies all interaction terms between the gravitational and ABJM sectors, given in (3.61) and (3.62) for the bosonic and fermionic contributions respectively. The same \bar{g}_M also defines $\Lambda = -\mu^2$ through (4.5).

Because of (4.10) gravity is weakly interacting as can be straightforwardly seen: One can define as usual the perturbative graviton $h_{\mu\nu}$ in terms of

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \kappa h_{\mu\nu} \quad (4.14)$$

so that its kinetic term is canonical²⁴ while the interactions, defined by κ , are weak. This is the case even though the background AdS₃ metric $g_{\mu\nu}^{(0)}$ satisfies an equation of motion involving the finite coupling \bar{g}_M in (3.42) and implying a finite cosmological constant. On the other hand, the pure ABJM part will couple to $h_{\mu\nu}$ only through κ .

Since $\kappa \rightarrow 0$ in the higgsing limit, the graviton $h_{\mu\nu}$ will be weakly coupled to matter. However, one still has a bit of freedom: By increasing \bar{g}_M it is possible to increase the coupling of $h_{\mu\nu}$ to (3.61) and (3.62), though in doing so one also increases the curvature of the background AdS space. If we instead keep \bar{g}_M of $\mathcal{O}(1)$, then the pure ABJM and interaction terms will be of the same order and both will only generate weak gravitational perturbations.

²³We have not established an exact dictionary between our conventions and those of [20] and can therefore not compare our definition of (p,q) to theirs.

²⁴That is, proportional to $\frac{1}{2} h \square h + \dots$, without any additional coupling in front.

4.3 Higgsing summary and an alternative procedure

It is possible to envision an alternative higgsing procedure that leads to a set of tunable coupling constants for the gauge theory as well as for the gravity sector of the theory. In order to see how this might happen, let us concisely summarize the main points of the higgsing process for the topologically gauged ABJM theory of [9] so that the possibility of the alternative procedure involving a slightly different set of parameters and limits becomes clear.

4.3.1 The higgsing procedure discussed so far

The higgsing discussed in this paper rely on the introduction of a new parameter²⁵ g_M in the conformal supergravity sector of the topologically gauged ABJM theory and lead to relations between the fundamental parameters of the theory and the final gravitational and Yang-Mills coupling constants.²⁶ Starting from the following schematically written part of the bosonic lagrangian where, however, all parameters appear explicitly,

$$L = \frac{1}{g_M^2} L_{CS(\omega)} - R|Z|^2 + \frac{1}{\lambda}(AdA + A^3) - DZD\bar{Z} - \lambda^2 V^{1-\text{trace}}(Z) - \lambda g_M^2 V^{2-\text{trace}}(Z) - g_M^4 V^{3-\text{trace}}(Z) , \quad (4.15)$$

where $\lambda = \frac{2\pi}{k}$ with k the Chern-Simons level, higgsing is straightforward and works as follows: First the vector potential A is divided into two fields A^+ and A^- in the above lagrangian which then appear through a term $\frac{1}{\lambda}A^-F^+$. Inserting the scalar VEV $\langle Z^4 \rangle = v$ into the scalar field kinetic term one obtains a term $v^2(A^-)^2$, which together with the previous result leads to the equation $v^2A^- \sim \frac{1}{\lambda}F^+$. Together these generate a proper Yang-Mills kinetic term, namely $\frac{1}{\lambda^2 v^2}(F^+)^2$, and hence we conclude that the Yang-Mills coupling constant is determined by

$$g_{YM}^2 \sim \lambda^2 v^2. \quad (4.16)$$

Since we want to keep g_{YM}^2 finite when the limits are taken, we find that these must involve

$$v \rightarrow \infty, \quad \lambda \rightarrow 0. \quad (4.17)$$

It is also clear from the $R|Z|^2 = v^2 R + \dots$ term in the lagrangian above that it is necessary to identify the gravitational coupling constant as

$$\kappa^2 \sim \frac{1}{v^2}, \quad (4.18)$$

which then goes to zero when taking the limits (4.17). The lagrangian now reads schematically, to lowest order in the expansion around the VEV and with z the deviation from the

²⁵This parameter plays a similar role to the level introduced in three-dimensional conformal gravity by Horne and Witten in [44].

²⁶For details how this is done see Section 3.2.2.

VEV,

$$\begin{aligned}
L = & \frac{1}{g_M^2} L_{CS(\omega)} - \frac{1}{\kappa^2} R + \frac{1}{g_{YM}^2} (F^+)^2 - DzD\bar{z} \\
& - (\lambda^2 v^2(z)^4 + \text{subleading}) - (\lambda g_M^2 v^3(z)^3 + \text{subleading}) \\
& - (g_M^4 v^6 + g_M^4 v^5(z) + g_M^4 v^4(z)^2 + \text{subleading}) ,
\end{aligned} \tag{4.19}$$

where the sub-leading terms become small in the $v \rightarrow \infty$ limit. For the single- and double-trace potential terms (second line above) one of course also needs to show that, in this limit, any contributions that would grow relative to the explicitly given Yang-Mills terms vanish. For the single-trace term of the original ABJM theory this was shown in [8], while for the double-trace terms this was demonstrated in Section 3.3 of this paper. As also seen there, the linear term in the triple-trace potential (last line) cancels against a term coming from the expansion of $R|Z|^2$. Finally, we identified the coefficients in front of the three potential terms and the gravitational Chern-Simons term and found

$$\begin{aligned}
L = & \frac{1}{\kappa^2 \mu} L_{CS(\omega)} - \frac{1}{\kappa^2} R + \frac{1}{g_{YM}^2} (F^+)^2 - DzD\bar{z} \\
& - (g_{YM}^2(z)^4 + \text{subleading}) - (\mu g_{YM}(z)^3 + \text{subleading}) \\
& - \left(\frac{1}{\kappa^2 \ell^2} + \dots + \text{subleading} \right) ,
\end{aligned} \tag{4.20}$$

where *subleading*²⁷ refers to positive powers of κ or $\frac{1}{v^2}$, and where

$$\mu \sim \frac{g_M^2}{\kappa^2} , \quad \ell = \frac{1}{\mu} \tag{4.21}$$

with ℓ the AdS radius as in the previous sections. This confirms the prediction of [9] that this theory is automatically sitting at a chiral point in the sense of Li, Song and Strominger [11].

4.3.2 An alternative higgsing procedure

We have thus far described the higgsing procedure for the interacting ABJM plus gravitational theory, presented as an extension of the M2 to D2 procedure of [2] for pure ABJM. In this context, the starting point was uniquely determined by the requirement to obtain a pure supergravity part that comes with the usual $\frac{1}{g_M^2}$ coefficient, as well as the wish to recover pure ABJM at $g_M = 0$, *i.e.* to have no factors of g_M in the pure ABJM part of the lagrangian.

However if one forgets for a moment about the M2-brane origin of the theory and its associated ABJM interpretation, and only considers higgsing some field theory coupled to gravity, it is possible to consider a slightly different starting point:

$$\begin{aligned}
L = & L_{CS(\omega)} - g_M^2 R|Z|^2 + \frac{g_M^2}{\lambda} (AdA + A^3) - g_M^2 DZD\bar{Z} \\
& - \lambda^2 g_M^2 V^{1-\text{trace}}(Z) - \lambda g_M^4 V^{2-\text{trace}}(Z) - g_M^6 V^{3-\text{trace}}(Z) ,
\end{aligned} \tag{4.22}$$

²⁷The terms displayed are the non-vanishing ones with lowest number of scalar fields. Terms with more scalar fields have thus fewer powers of the VEV v .

which is essentially the lagrangian used in the previous discussion multiplied by a factor of g_M^2 . As we will see below this minor alteration nevertheless has an impact on the behavior of the various coefficients when taking the limits.

It can be easily verified that performing the higgsing based on the same VEV as above, $\langle Z^4 \rangle = v$, now leads to the identifications

$$g_{YM}^2 \sim \frac{\lambda^2 v^2}{g_M^2}, \quad \kappa^2 \sim \frac{1}{v^2 g_M^2}, \quad (4.23)$$

resulting in the lagrangian

$$\begin{aligned} L = & L_{CS(\omega)} - \frac{1}{\kappa^2} R + \frac{1}{g_{YM}^2} (F^+)^2 - g_M^2 D z D \bar{z} \\ & - (\lambda^2 v^2 g_M^2 (z)^4 + \text{subleading}) - (\lambda g_M^4 v^3 (z)^3 + \text{subleading}) \\ & - (g_M^6 v^6 + \dots + \text{subleading}) . \end{aligned} \quad (4.24)$$

At this point one may rescale the scalar fields

$$z = \frac{\tilde{z}}{g_{YM} g_M}, \quad (4.25)$$

which will turn all coefficients in L into combinations of only κ and g_{YM} :

$$\begin{aligned} L = & L_{CS(\omega)} - \frac{1}{\kappa^2} R + \frac{1}{g_{YM}^2} (F^+)^2 - \frac{1}{g_{YM}^2} D \tilde{z} D \bar{\tilde{z}} \\ & - \left(\frac{1}{g_{YM}^2} (\tilde{z})^4 + \text{subleading} \right) - \left(\frac{1}{\kappa^2 g_{YM}^2} (\tilde{z})^3 + \text{subleading} \right) \\ & - \left(\frac{1}{\kappa^6} + \frac{1}{\kappa^5 g_{YM}} (\tilde{z}) + \frac{1}{\kappa^4 g_{YM}^2} (\tilde{z})^2 + \text{subleading} \right) . \end{aligned} \quad (4.26)$$

From this form of the lagrangian we conclude that

$$\mu = \kappa^{-2}, \quad \ell = \kappa^2 \quad (4.27)$$

and hence that the theory is at the ‘chiral point’ just as we found in the first analysis.

One also discovers that the sub-leading terms all appear with more than two inverse powers of g_{YM} and thus become negligible close to the IR fixed point, *i.e.* for large Yang-Mills coupling. This is not the same procedure as that done in the pure ABJM case, where we wanted $v \rightarrow \infty$, $\lambda \rightarrow 0$ and we obtained three-dimensional SYM at *finite* $g_{YM} \sim v\lambda$. However, the current starting point may not have an M2/ABJM interpretation, in which case this is not an issue.

The analysis of the sub-leading terms is done as follows: The one-trace (ABJM) potential V (proportional to Z^6) has an expansion around the VEV v that gives the term at order $v^2 \tilde{z}^4 / g_{YM}^2$. Replacing one \tilde{z} with a VEV v means acquiring an extra factor of $v g_M g_{YM} = \frac{g_{YM}}{\kappa}$. As is known from [2, 33] this and the terms resulting after two and more such replacements all vanish. Replacing a VEV by \tilde{z} on the other hand will produce extra factors of g_{YM} in the denominator and yield sub-leading expressions in the IR limit.

For the double-trace potential we get a cubic contribution with the remaining terms in its expansion behaving in a similar way. Finally, the triple-trace potential starts with the

cosmological constant term κ^{-6} and by replacing VEVs with \tilde{z} 's one gets a linear term that vanishes and a bilinear contribution that is a mass term with coefficient $g_{YM}^{-2}\kappa^{-4}$. One also has a term linear in the scalar field with a non-vanishing coefficient. However, this term is needed since it will cancel a similar one coming from the expansion of the conformal $R|Z|^2$ in the lagrangian. The rest of the terms are all sub-leading in the $g_{YM} \rightarrow \infty$ limit.

The main novelty of this alternative approach is that the three limits

$$v \rightarrow \infty, \lambda \rightarrow 0, g_M \rightarrow 0 \quad (4.28)$$

can be used to keep the gravitational coupling

$$\kappa^2 = \frac{8}{g_M^2 v^2} \quad (4.29)$$

fixed and finite. However, it does not seem possible to completely switch off gravity since then the AdS radius goes to zero, *i.e.* in that case one is dealing with a background of infinite curvature. We should keep in mind that in this case the M2/ABJM interpretation may be lost but one can nevertheless study the model in its own right.

Hence the two pictures present different virtues: While for the usual ABJM interpretation, considered in the bulk of this paper, we could not avoid to decouple gravity (since $v \sim \frac{1}{\kappa}$ and $v \rightarrow \infty$). This, however, happens at a fixed AdS radius, which is welcome. On the other hand, in the alternative approach we have a fixed gravitational coupling constant with an AdS radius that is proportional to the gravitational coupling. Both features are somewhat unusual but should be viewed as consequences of the fact that the theory is tied to the chiral point. It seems that the physical effects of the chiral point in this model need to be further studied.

5. Conclusions and Outlook

In this paper we analyzed the higgsing [2] of the topologically gauged ABJM model constructed in [9]. We first re-expressed the model of [9] in a bifundamental Lie algebra formulation and introduced a coupling constant g_M for the conformal supergravity sector and the matter-gravity interactions. We proceeded to briefly review the higgsing for the original ABJM case. In doing so we took some care to clarify the role of U(1) factors that were previously omitted in the literature.

We then presented the higgsing procedure for the theory of [9] as an extension to ABJM. For the latter one needs to take the VEV v and CS level k to infinity, with $\frac{v}{k} \equiv g_{YM}$ fixed, while in the theory studied in this paper one must also take $g_M \rightarrow 0$, with $\frac{g_M v^2 N}{8} \equiv \mu$ fixed, where $\frac{1}{\mu}$ is the coefficient of the gravitational Chern-Simons term. This leads to an $\mathcal{N} = 6$ supersymmetric theory involving an AdS_3 background of finite radius ℓ with $\mu\ell = 1$. All matter coupled to gravity through a weak Newton coupling, $\kappa^2 \sim \frac{1}{v^2} \rightarrow 0$.

The corresponding action was given in Eq. (1.1), with the gravity part being a supersymmetric generalization of topologically massive gravity (TMG) at the ‘chiral point’ in

the sense of [11]. The supersymmetries appearing in the higgsed theory can be seen to arise as linear combinations of supersymmetries and special superconformal symmetries of the unhiggsed theory. One should note, however, that we here find the sign of the Einstein-Hilbert term reversed compared to [11] as is more customary in the three-dimensional TMG literature. It would be interesting to further elucidate the role of this sign issue and to see whether our construction can provide a different angle on the subtle questions of gravitational boundary conditions that determine the nature of the physical states and the dual CFT for these theories.

Besides the standard ABJM Higgs effect we also find that the R-symmetry is broken to $SU(3)$ with the result that all but one of the eight Goldstone scalars X_0^I are ‘eaten’ by gauge fields. This makes it hard to find a D-brane interpretation for the final theory unless the branes experience some kind of position stabilization in analogy to the discussion of [45] in the context of brane-world models. Since in string theory independent world-volume Einstein gravity on branes is not allowed, world-volume gravity would most likely arise from the space-time fields. Indeed, in brane-world models, world-volume gravity does arise from brane-restriction of space-time gravity as in the Randall-Sundrum model [46], and position stabilization generates an effective action for the space-time fields restricted to the world-volume [45].

Similar questions remain in connection with the interpretation of the theory in the conformal phase. Since the ABJM theory captures the dynamics of N M2-branes on a $\mathbb{C}^4/\mathbb{Z}_k$ orbifold, the coupling to conformal world-volume supergravity might be thought of as a construction resembling the string action with world-sheet gravity, used *e.g.* when quantizing the string on higher genus Riemann surfaces, or perhaps as a first step towards coupling the theory to more general backgrounds.

In this context one could also ask what the string/M-theory origin of a topologically gauged theory might be. There exist examples for which the low energy limit has been known to fail in decoupling gravity, *e.g.* systems involving D3-branes intersecting D7/O7 configurations on a 1+1d subspace. This special property is due to the particular way α' appears in front of the different terms in the lagrangian [47, 48]. Perhaps the model of [9] arises in a similar way from some yet to be determined M-theory configuration. It would be interesting to investigate further if such a picture can be realized.

Acknowledgments

We would like to thank Bobby Ezhuthachan, Alex Maloney, Andy Royston and Kostas Skenderis for discussions and comments. CP is supported by the STFC grant ST/G000395/1 and XC by the Belgian Fonds de la Recherche Scientifique (FNRS) and Belgian Science Policy IAP.

A. Expansion and trace formulas

In this appendix we list various formulas that were used for the higgsing of the topologically gauged ABJM action. In particular, we find for the bosonic single-trace 2-scalar structures:

$$\begin{aligned} |Z|^2 &= \sum_A \text{tr}(Z^A Z_A^\dagger) = \text{tr}\left(v^2 + v(z^4 + z_4^\dagger) + z^A z_A^\dagger\right) \\ \text{tr}(Z^B Z_A^\dagger) &= \text{tr}\left(v^2 \delta_{A,4} \delta^{B,4} + v(z^B \delta_{A,4} + z_A^\dagger \delta^{B,4}) + z^B z_A^\dagger\right) \\ \text{tr}(Z^B \tilde{D}_\rho Z_A^\dagger) &= \text{tr}\left(v \tilde{D}_\rho v \delta_{A,4} \delta^{B,4} + z^B \tilde{D}_\rho v \delta_{A,4} + v \tilde{D}_\rho z_A^\dagger \delta^{B,4} + z^B \tilde{D}_\rho z_A^\dagger\right), \end{aligned} \quad (\text{A.1})$$

which then also implies

$$\begin{aligned} \text{tr}(Z^B Z_A^\dagger) \text{tr}(Z^A Z_B^\dagger) &= (v^2 N + v \text{tr}(z^4 + z_4^\dagger))^2 + 2v^2 N \text{tr}(z^4 z_4^\dagger) \\ &\quad + 2v^2 (\text{tr} z^A \text{tr} z_A^\dagger - \text{tr} z^4 \text{tr} z_4^\dagger) \\ &\quad + 2v (\text{tr} z^A \text{tr}(z^4 z_A^\dagger) + \text{tr} z_A^\dagger \text{tr}(z_4^\dagger z^A)) + \text{tr}(z^A z_B^\dagger) \text{tr}(z^B z_A^\dagger) \\ \text{tr}(Z^A Z_C^\dagger) \text{tr}(Z^B Z_A^\dagger) \text{tr}(Z^C Z_B^\dagger) &= (v^2 N + v \text{tr}(z^4 + z_4^\dagger))^3 + 3v^2 (v^2 N + v \text{tr}(z^4 + z_4^\dagger)) \times \\ &\quad \times (\text{tr} z^A \text{tr} z_A^\dagger - \text{tr} z^4 \text{tr} z_4^\dagger) + 3v^4 N^2 \text{tr}(z^4 + z_4^\dagger) \times \\ &\quad + 3v^3 N [\text{tr}(z^4 + z_4^\dagger) \text{tr}(z^4 z_4^\dagger) + \text{tr}(z^4 z_A^\dagger) \text{tr} z^A \\ &\quad + \text{tr}(z^A z_4^\dagger) \text{tr} z_4^\dagger] + \mathcal{O}(v^2). \end{aligned} \quad (\text{A.2})$$

The bosonic single-trace 4-scalar structures are

$$\begin{aligned} \text{tr}(Z_D^\dagger Z^B Z_C^\dagger Z^C - Z^B Z_D^\dagger Z^C Z_C^\dagger) &= \text{tr}\left(v[z_C^\dagger, z^C](\delta^{B,4} z_D^\dagger + \delta_{D,4} z^B) + v z_D^\dagger([z^B, z^4] + [z^B, z_4^\dagger])\right. \\ &\quad \left.+ z_D^\dagger z^B z_C^\dagger z^C - z^B z_D^\dagger z^C z_C^\dagger\right) \\ \text{tr}(Z_D^\dagger Z^D Z_C^\dagger Z^C - Z^D Z_D^\dagger Z^C Z_C^\dagger) &= \text{tr}\left(2v(z^4 + z_4^\dagger)[z_D^\dagger, z^D] + z_D^\dagger z^D z_C^\dagger z^C - z^D z_D^\dagger z^C z_C^\dagger\right). \end{aligned} \quad (\text{A.3})$$

Combining the above we get

$$\begin{aligned} \text{tr}(Z^D Z_B^\dagger) \text{tr}(Z_D^\dagger Z^B Z_C^\dagger Z^C - Z^B Z_D^\dagger Z^C Z_C^\dagger) &= \text{tr}(v^2) \text{tr}\left(v[z_C^\dagger, z^C](z_4^\dagger + z^4) + v(z^4 + z_4^\dagger)[z_4^\dagger, z^4]\right) \\ &\quad + \text{tr}(v^2) \text{tr}(z_4^\dagger z^4 z_C^\dagger z^C - z^4 z_4^\dagger z^C z_C^\dagger) \\ &\quad + \text{tr}(v z_D^\dagger) \text{tr}\left(v[z_C^\dagger, z^C] z_D^\dagger + v z_D^\dagger [z^4, z_4^\dagger]\right) \\ &\quad + \text{tr}(v z_B^\dagger) \text{tr}\left(v[z_C^\dagger, z^C] z^B + v z^B [z^4, z_4^\dagger]\right) \\ &\quad + \text{tr}(v z^4) \text{tr}\left(v[z_C^\dagger, z^C] z^4\right) \\ &\quad + \text{tr}(v z_4^\dagger) \text{tr}(v[z_C^\dagger, z^C] z_4^\dagger) \\ &\quad + \text{tr}(z^D z_B^\dagger) \text{tr}\left(v z_D^\dagger([z^B, z^4] + [z^B, z_4^\dagger])\right) \\ &\quad + \text{tr}(z^D z_4^\dagger) \text{tr}\left(v[z_C^\dagger, z^C] z_D^\dagger\right) \\ &\quad + \text{tr}(z^4 z_B^\dagger) \text{tr}(v[z_C^\dagger, z^C] z^B) \\ &\quad + \text{tr}(v z^A) \text{tr}(z_A^\dagger z^4 z_C^\dagger z^C - z^4 z_A^\dagger z^C z_C^\dagger) \\ &\quad + \text{tr}(v z_A^\dagger) \text{tr}(z_4^\dagger z^A z_C^\dagger z^C - z^A z_4^\dagger z^C z_C^\dagger) \\ &\quad + \text{tr}(z^D z_B^\dagger) \text{tr}(z_D^\dagger z^B z_C^\dagger z^C - z^B z_D^\dagger z^C z_C^\dagger). \end{aligned} \quad (\text{A.4})$$

Finally we get the combinations:

$$\begin{aligned}
|Z^6| &= \text{tr}(v^2)^3 + \text{tr}(v(z^4 + z_4^\dagger))^3 + \text{tr}(z^A z_A^\dagger)^3 \\
&\quad + 3 \left(\text{tr}(v^2)^2 [\text{tr}(v(z^4 + z_4^\dagger)) + \text{tr}(z^A z_A^\dagger)] \right. \\
&\quad + \text{tr}(v(z^4 + z_4^\dagger))^2 [\text{tr}(v^2) + \text{tr}(z^A z_A^\dagger)] \\
&\quad + \left. \text{tr}(z^A z_A^\dagger)^2 [\text{tr}(v^2) + \text{tr}(v(z^4 + z_4^\dagger))] \right) \\
&\quad + 6 \text{tr}(v^2) \text{tr}(v(z^4 + z_4^\dagger)) \text{tr}(z^A z_A^\dagger) \tag{A.5}
\end{aligned}$$

$$\begin{aligned}
|Z^2| \text{tr}(Z^A Z_B^\dagger) \text{tr}(Z^B Z_A^\dagger) &= \text{tr}(v^2)^3 + \text{tr}(v^2)^2 \left(3 \text{tr}(v z^4) + 3 \text{tr}(v z_4^\dagger) \right. \\
&\quad \left. + \text{tr}(z^A z_A^\dagger) + 2 \text{tr}(z^4 z_4^\dagger) \right) \\
&\quad + \text{tr}(v^2) \left(3(\text{tr}(v z^4) + \text{tr}(v z_4^\dagger))^2 \right. \\
&\quad + 2 \text{tr}(v z_A^\dagger) \text{tr}(z^A z_4^\dagger) + \text{tr}(z^A z_B^\dagger) \text{tr}(z^B z_A^\dagger) \\
&\quad + 2 \text{tr}(z^4 z_4^\dagger) \text{tr}(v z^4) + 2 \text{tr}(z^4 z_4^\dagger) \text{tr}(v z_4^\dagger) \\
&\quad + 2 \text{tr}(v z^A) \text{tr}(z^4 z_A^\dagger) + \text{tr}(z^4 z_4^\dagger) \text{tr}(z^A z_A^\dagger) \\
&\quad + \left. 2 \text{tr}(v z^4) \text{tr}(z^A z_A^\dagger) + 2 \text{tr}(v z_4^\dagger) \text{tr}(z^A z_A^\dagger) \right) \\
&\quad + \text{tr}(z^C z_C^\dagger) \text{tr}(z^A z_B^\dagger) \text{tr}(z^B z_A^\dagger) + \left(\text{tr}(v z^4) + \text{tr}(v z_4^\dagger) \right)^3 \\
&\quad + 2 \left(\text{tr}(v z^A) \text{tr}(z^4 z_A^\dagger) + \text{tr}(v z_A^\dagger) \text{tr}(z^A z_4^\dagger) \right) \times \\
&\quad \times \left(\text{tr}(v z^4) + \text{tr}(v z_4^\dagger) + \text{tr}(z^A z_A^\dagger) \right) \\
&\quad + \text{tr}(z^A z_A^\dagger) \left(\text{tr}(v z^4) + \text{tr}(v z_4^\dagger) \right)^2 \\
&\quad + \text{tr}(z^A z_B^\dagger) \text{tr}(z^B z_A^\dagger) \text{tr}(v z^4) \\
&\quad + \left. \text{tr}(z^A z_B^\dagger) \text{tr}(z^B z_A^\dagger) \text{tr}(v z_4^\dagger) \right) \tag{A.6}
\end{aligned}$$

$$\begin{aligned}
|Z^2| \text{tr}(Z_D^\dagger Z^D Z_C^\dagger Z^C - Z^D Z_D^\dagger Z^C Z_C^\dagger) &= \left(2 \text{tr}(v(z^4 + z_4^\dagger)[z_D^\dagger, z^D]) \right. \\
&\quad \left. + \text{tr}(z_D^\dagger z^D z_C^\dagger z^C - z^D z_D^\dagger z^C z_C^\dagger) \right) \\
&\quad \times \left(\text{tr}(v^2) + \text{tr}(v(z^4 + z_4^\dagger)) + \text{tr}(z^A z_A^\dagger) \right). \tag{A.7}
\end{aligned}$$

Working with the normalisation $T^0 = \mathbb{1}_{N \times N}$, considering $[T^a, T^b] = i f^{ab}{}_c T^c$ and $\text{tr}(\{T^a, T^b\} T^c) = d^{abc}$, a few trace formulas relevant for the bosonic terms are

$$\begin{aligned}
[z^A, z_A^\dagger] &= -X_a^{A+4} X_b^A f^{ab}{}_c T^c \\
\text{tr}(z^4 + z_4^\dagger) &= N \sqrt{2} X_0^4 \\
\text{tr}(z^4 - z_4^\dagger) &= i \sqrt{2} N X_0^8 \\
\text{tr}(z^A) \text{tr}(z_A^\dagger) - \text{tr}(z^4) \text{tr}(z_4^\dagger) &= \frac{N^2}{2} \left(X_0^I X_0^I - X_0^4 X_0^4 - X_0^8 X_0^8 \right) \\
z^A z_A^\dagger &= \frac{1}{2} \left(X_0^I X_0^I + X_a^I X_b^I T^a T^b + 2 X_a^A X_0^{A+4} T^a - 2 X_0^A X_a^{A+4} T^a - X_a^{A+4} X_b^A f^{ab}{}_c T^c \right) \\
z_A^\dagger z^A &= \frac{1}{2} \left(X_0^I X_0^I + X_a^I X_b^I T^a T^b + 2 X_a^A X_0^{A+4} T^a - 2 X_0^A X_a^{A+4} T^a + X_a^{A+4} X_b^A f^{ab}{}_c T^c \right) \\
\text{tr}(z^A z_A^\dagger) &= \frac{1}{2} (N X_0^I X_0^I + X_a^I X_a^I) \\
\text{tr}((z^4 + z_4^\dagger)[z_D^\dagger, z^D]) &= -\sqrt{2} f^{abc} X_a^{A+4} X_b^A X_c^8
\end{aligned}$$

$$\begin{aligned}
\text{tr}(z_4^\dagger[z^4, z_4^\dagger]) &= \frac{1}{\sqrt{2}} f^{abc} X_a^8 X_b^4 (X_c^8 + iX_c^4) \\
\text{tr}((z^4 - z_4^\dagger)[z_D^\dagger, z^D]) &= i\sqrt{2} f^{abc} X_a^{A+4} X_b^A X_c^4 \\
\text{tr}(z_D^\dagger z^D z_C^\dagger z^C - z^D z_D^\dagger z^C z_C^\dagger) &= X_a^{A+4} X_b^A f^{abc} (2X_c^B X_0^{B+4} - 2X_0^B X_c^{B+4} + \frac{1}{2} d^{ef}{}_c X_e^I X_f^I) \\
\text{tr}(z_4^\dagger z^4 z_C^\dagger z^C - z^4 z_4^\dagger z^C z_C^\dagger) &= \frac{1}{2} f^{abc} \left\{ X_a^{A+4} X_b^A (2X_c^4 X_0^8 - 2X_0^4 X_c^8 + \frac{1}{2} d^{ed}{}_c (X_e^4 X_f^4 + X_e^8 X_f^8)) \right. \\
&\quad \left. + X_a^8 X_b^4 (2X_c^B X_0^{B+4} - 2X_0^B X_c^{B+4} + \frac{1}{2} d^{ef}{}_c (X_e^I X_f^I)) \right\} \\
\text{tr}(z^D) \text{tr}(z_D^\dagger[z_C^\dagger, z^C]) &= -\frac{N}{2} f^{abc} X_a^{C+4} X_b^C (X_c^{D+4} + iX_c^D)(X_0^D + iX_0^{D+4}) \\
\text{tr}(z^D) \text{tr}(z_D^\dagger[z^4, z_4^\dagger]) &= \frac{N}{2} f^{abc} X_a^8 X_b^4 (X_c^{D+4} + iX_c^D)(X_0^D + iX_0^{D+4}) \\
\text{tr}(z_D^\dagger) \text{tr}(z^D[z_C^\dagger, z^C]) &= -\frac{N}{2} f^{abc} X_a^{C+4} X_b^C (X_c^{D+4} - iX_c^D)(X_0^D - iX_0^{D+4}) \\
\text{tr}(z_D^\dagger) \text{tr}(z^D[z^4, z_4^\dagger]) &= \frac{N}{2} f^{abc} X_a^8 X_b^4 (X_c^{D+4} - iX_c^D)(X_0^D - iX_0^{D+4}) \\
\text{tr}(z^A) \text{tr}(z_A^\dagger) \text{tr}(z^4 z_4^\dagger) &= \frac{N^2}{4} X_0^I X_0^I \left[N((X_0^4)^2 + (X_0^8)^2) + (X_a^4)^2 + (X_a^8)^2 \right] \\
\text{tr}(z^A) \text{tr}(z_4^\dagger) \text{tr}(z^4 z_A^\dagger) &= \frac{N^2}{4} \left[N X_0^I X_0^I ((X_0^4)^2 + (X_0^8)^2) + (X_0^A X_0^4 + X_0^{A+4} X_0^8)(X_a^A X_a^4 + X_a^{A+4} X_a^8) \right. \\
&\quad \left. + (X_0^A X_0^8 - X_0^{A+4} X_0^4)(X_a^A X_a^8 - X_a^{A+4} X_a^4) \right] \\
\text{tr}(z_A^\dagger) \text{tr}(z^4) \text{tr}(z^A z_4^\dagger) &= \frac{N^2}{4} \left[N X_0^I X_0^I ((X_0^4)^2 + (X_0^8)^2) \right. \\
&\quad \left. + (X_0^A - iX_0^{A+4})(iX_a^A - X_a^{A+4})(X_0^4 + iX_0^8)(-iX_a^4 - X_a^8) \right] \\
&= \text{tr}(z^A) \text{tr}(z_4^\dagger) \text{tr}(z^4 z_A^\dagger) \\
\text{tr}(z_A^\dagger) \text{tr}(z^B) \text{tr}(z^A z_B^\dagger) &= \frac{N^2}{4} \left[N(X_0^I X_0^I)^2 + (X_0^A X_0^B + X_0^{A+4} X_0^{B+4})(X_a^A X_a^B + X_a^{A+4} X_a^{B+4}) \right. \\
&\quad \left. + (X_0^A X_0^{B+4} - X_0^{A+4} X_0^B)(X_a^A X_a^{B+4} - X_a^{A+4} X_a^B) \right] \\
\text{tr}(z^A z_B^\dagger) &= \frac{1}{2} \left(N(X_0^A + iX_0^{A+4})(X_0^B - iX_0^{B+4}) - (iX_a^A - X_a^{A+4})(X_a^{B+4} + iX_a^B) \right) \\
\text{tr}(z^A z_B^\dagger) \text{tr}(z^B z_A^\dagger) &= \frac{1}{4} \left[N^2 (X_0^I X_0^I)^2 + (X_a^I X_b^I)^2 + (X_a^A X_b^{A+4} - X_a^{A+4} X_b^A)^2 \right. \\
&\quad \left. + 2N[(X_0^I X_a^I)^2 + (X_0^{A+4} X_a^A - X_0^A X_a^{A+4})^2] \right] \\
\text{tr}(z^A z_4^\dagger) \text{tr}(z^4 z_A^\dagger) &= \frac{1}{4} \left[N^2 \left(X_0^I X_0^I (X_0^4 X_0^4 + X_0^8 X_0^8) \right) + |(X_a^8 + iX_a^4)(iX_a^A - X_a^{A+4})|^2 \right. \\
&\quad \left. - N(X_0^4 - iX_0^8)(X_0^A + iX_0^{A+4})(X_a^{A+4} + iX_a^A)(iX_a^4 - X_a^8) - c.c. \right] \\
&= \frac{1}{4} \left[N^2 X_0^I X_0^I (X_0^4 X_0^4 + X_0^8 X_0^8) \right. \\
&\quad + (X_a^{A+4} X_b^4 + X_a^A X_b^8)(X_b^{A+4} X_a^4 + X_b^A X_a^8) \\
&\quad + (X_a^A X_b^4 - X_a^{A+4} X_b^8)(X_b^A X_a^4 - X_b^{A+4} X_a^8) \\
&\quad + 2N[(X_0^A X_0^4 + X_0^{A+4} X_0^8)(X_a^{A+4} X_a^8 + X_a^A X_a^4) \\
&\quad \left. + (X_0^4 X_0^{A+4} - X_0^8 X_0^A)(X_a^{A+4} X_a^4 - X_a^A X_a^8) \right] \\
\text{tr}(z^4 z_4^\dagger) \text{tr}(z^A z_A^\dagger) &= \frac{1}{4} (N X_0^4 X_0^4 + N X_0^8 X_0^8 + X_a^4 X_a^4 + X_a^8 X_a^8)(N X_0^I X_0^I + X_a^I X_a^I), \tag{A.8}
\end{aligned}$$

where in the first identity we do not sum over A .

Formulas relevant for the fermionic terms are

$$\begin{aligned}
[z^A, z_D^\dagger] &= \frac{1}{2} f^{ab} (iX_a^A X_b^D - X_a^{A+4} X_b^D + X_a^A X_b^{D+4} + iX_a^{A+4} X_b^{D+4}) T^c \\
[z^A, z^D] &= \frac{1}{2} f^{ab} (-iX_a^A X_b^D + X_a^{A+4} X_b^D + X_a^A X_b^{D+4} + iX_a^{A+4} X_b^{D+4}) T^c \\
\text{tr}(Z_D^\dagger Z^C Z_C^\dagger Z^D - Z_D^\dagger Z^D Z_C^\dagger Z^C) &= 2\sqrt{2} v f^{abc} X_a^{A+4} X_b^A X_c^8 - f^{abc} X_a^{A+4} X_b^A \\
&\quad (2X_c^B X_0^{B+4} - 2X_0^B X_c^{B+4} + \frac{1}{2} d^{ef} X_e^I X_f^I) \\
\text{tr}(Z^B Z_A^\dagger) &= v^2 N \delta_{A,4} \delta^{B,4} + \frac{vN}{\sqrt{2}} ((X_0^A - iX_0^{A+4}) \delta^{B,4} + \delta_{A,4} (X_0^B + iX_0^{B+4})) \\
&\quad + \frac{N}{2} (X_0^B + iX_0^{B+4}) (X_0^A - iX_0^{A+4}) + \frac{1}{2} (X_a^{B+4} - iX_a^B) (X_a^{A+4} + iX_a^A) \\
\text{tr}(Z^B \tilde{D}_\rho Z_A^\dagger) &= \frac{vN}{\sqrt{2}} \delta^{B,4} \partial_\rho (X_0^A - iX_0^{A+4}) + \sqrt{2} v (X_a^B + iX_a^{B+4}) A_\rho^a - \delta^{A,4} \\
&\quad + \frac{N}{2} (X_0^B + iX_0^{B+4}) \partial_\rho (X_0^A - iX_0^{A+4}) + \frac{1}{2} (X_a^{B+4} - iX_a^B) D_\rho (X_a^{A+4} + iX_a^A) \\
&\quad + \text{subleading} - iN \delta^{A,4} A_{\mu 0}^- \left[2v^2 \delta_{B,4} + \sqrt{2} (X_0^B + iX_0^{B+4}) \right] \\
\text{tr}(Z_{[D}^\dagger Z^{[A} Z_{C]}^\dagger Z^{B]} - Z_{[D}^\dagger Z^{B]} Z_{C]}^\dagger Z^A]) &= \frac{1}{\sqrt{2}} v i f^{abc} \left[(iX_a^{[C} + X_a^{[C+4]})(iX_b^{D]} + X_b^{D]+4}) \times \right. \\
&\quad \times (iX_c^{[A} - X_c^{[A+4]}) \delta^{B],4} \\
&\quad \left. + (iX_a^{[A} - X_a^{[A+4]})(iX_b^{B]} - X_b^{B]+4})(iX_c^{[C} + X_c^{[C+4]}) \delta^{D],4} \right] \\
&\quad + \text{subleading} \\
\text{tr}(\bar{\Psi}_D^\dagger \Psi^B) &= \frac{N}{\sqrt{2}} (\psi_0^{[A} + i\psi_0^{[A+4]}) \partial_\mu (X_0^{B]} - iX_0^{B]+4}) \\
&\quad + (i\psi_a^{[A} - \psi_a^{[A+4]}) \left[\frac{D_\mu (-iX_a^{B]} - X_a^{B]+4})}{\sqrt{2}} - 2iv A_{\mu a}^- \delta^{B,4} \right] \\
&\quad - 2iN v A_{\mu 0}^- \delta^{A,4} (\psi_0^A + i\psi_0^{A+4}) + \text{subleading} \\
\text{tr}(\Psi_A \Psi^{\dagger C}) &= N (\psi_0^A + i\psi_0^{A+4}) (\psi_0^{\dagger C} - i\psi_0^{\dagger C+4}) + (i\psi_a^A - \psi_a^{A+4}) (-i\psi_a^{\dagger C} - \psi_a^{C+4}) \\
\text{tr}(\Psi_A Z_B^\dagger) &= \frac{N}{\sqrt{2}} (\psi_0^A + i\psi_0^{A+4}) (X_0^B - iX_0^{B+4}) \\
&\quad + \frac{1}{\sqrt{2}} (i\psi_a^A - \psi_a^{A+4}) (-iX_a^B - X_a^{B+4}) + vN \delta_{B,4} (\psi_0^A + i\psi_0^{A+4}) \\
\text{tr}(\Psi^B (Z^D Z_D^\dagger Z^A - Z^A Z_D^\dagger Z^D)) &= -iv (i\psi_c^B - \psi_c^{B+4}) f^{abc} X_a^8 (iX_b^A - X_b^{A+4}) \\
&\quad - v \delta^{A,4} X_a^{D+4} X_b^D (i\psi_c^B - \psi_c^{B+4}) + \text{subleading} \\
\text{tr}(\Psi^D (Z^{[B} Z_D^\dagger Z^{A]} - Z^{[A} Z_D^\dagger Z^{B]})) &= -\frac{i}{2} v (i\psi_c^A - \psi_c^8) (iX_a^{[A} - X_a^{[A+4]})(iX_b^{B]} - X_b^{B]+4}) f^{abc} \\
&\quad - iv (i\psi_c^D - \psi_c^{D+4}) \delta^{A,4} (-iX_{a,D} - X_{a,D+4}) (iX_b^{B]} - X_b^{B]+4}) f^{abc} \\
&\quad + \text{subleading} . \tag{A.9}
\end{aligned}$$

B. Subleading terms in v

Here we present formulas for the Higgsed action down to $\mathcal{O}(v)$ for both the bosonic and fermionic pieces.

B.1 Bosonic terms

The bosonic 2-trace terms give

$$\begin{aligned}
-V_{2-trace,v^2} = & -\frac{g_M^2 v^2 N \lambda}{8} X_a^{A+4} X_b^A f^{abc} (2X_c^B X_0^{B+4} - 2X_0^B X_c^{B+4} + \frac{\lambda}{2} d^{ef}{}_c X_e^I X_f^I) \\
& + \frac{g_M^2 v^2 N \lambda}{4} f^{abc} \left\{ X_a^{A+4} X_b^A (2X_c^4 X_0^8 - 2X_0^4 X_c^8 + \frac{1}{2} d^{ed}{}_c (X_e^4 X_f^4 + X_e^8 X_f^8)) \right. \\
& + X_a^8 X_b^4 (2X_c^B X_0^{B+4} - 2X_0^B X_c^{B+4} + \frac{1}{2} d^{ef}{}_c (X_e^I X_f^I)) \left. \right\} \\
& - \frac{g_M^2 v^2 N \lambda}{2} f^{abc} X_a^{A+4} X_b^A X_c^4 X_0^8 \\
& + \frac{g_M^2 v^2 N \lambda}{2} f^{abc} (X_c^{D+4} X_0^D - X_c^D X_0^{D+4}) (X_a^8 X_b^4 - X_a^{C+4} X_b^C). \quad (B.1)
\end{aligned}$$

The bosonic 3-trace terms give

$$\begin{aligned}
-V_{3-trace,v^2} = & \frac{5g_M^4 v^2 N}{4 \cdot 256} (N^2 (X_0^I X_0^I)^2 + (X_a^I X_a^I)^2 + 2X_0^I X_0^I X_a^J X_a^J) \\
& - \frac{g_M^4 v^2 N}{2 \cdot 64} \left[N^2 X_0^I X_0^I (X_0^J X_0^J - (X_0^4)^2 - (X_0^8)^2) + N[2(X_0^I X_a^I)^2 \right. \\
& + 2(X_0^{A+4} X_a^A - X_0^A X_a^{A+4})^2 + X_0^I X_0^I ((X_a^4)^2 + (X_a^8)^2) \\
& + X_a^I X_a^I ((X_0^4)^2 + (X_0^8)^2) - 4(X_0^A X_0^4 + X_0^{A+4} X_0^8) (X_a^{A+4} X_a^8 + X_a^A X_a^4) \\
& - 4(X_0^4 X_0^{A+4} - X_0^8 X_0^A) (X_a^{A+4} X_a^4 - X_a^A X_a^8) \left. \right] + X_a^I X_a^I ((X_b^4)^2 + (X_b^8)^2) \\
& - 2(X_a^{A+4} X_b^4 + X_a^A X_b^8) (X_b^{A+4} X_b^4 + X_b^A X_b^8) + (X_a^I X_b^I)^2 \\
& - 2(X_a^A X_b^4 - X_a^{A+4} X_b^8) (X_b^A X_a^4 - X_b^{A+4} X_a^8) + (X_a^A X_b^{A+4} - X_a^{A+4} X_b^A)^2 \left. \right] \\
& + \frac{g_M^4 v^2 N^2}{64} \left[N X_0^I X_0^I (X_0^J X_0^J - (X_0^4)^2 - (X_0^8)^2) + X_0^I X_0^I ((X_a^4)^2 + (X_a^8)^2) \right. \\
& + (X_0^A X_0^B + X_0^{A+4} X_0^{B+4}) (X_a^A X_a^B + X_a^{A+4} X_a^{B+4}) \\
& + (X_0^A X_0^{B+4} - X_0^{A+4} X_0^B) (X_a^A X_a^{B+4} - X_a^{A+4} X_a^B) \\
& - 2(X_0^A X_0^4 + X_0^{A+4} X_0^8) (X_a^A X_a^4 + X_a^{A+4} X_a^8) \\
& \left. - 2(X_0^A X_0^8 - X_0^{A+4} X_0^4) (X_a^A X_a^8 - X_a^{A+4} X_a^4) \right]. \quad (B.2)
\end{aligned}$$

Note that if we exclude the contributions containing X_0 's, which decoupled during the higgsing of the pure ABJM theory, we are left with

$$\begin{aligned}
-V_{v^2}^{\text{SU}(N)} = & -\frac{g_M^2 v^3 N \lambda}{2\sqrt{2}} f^{abc} X_a^{A+4} X_b^A X_c^8 \\
& + \frac{g_M^2 v^2 N \lambda}{8} f^{abc} d^{ef}{}_c \left[X_a^{A+4} X_b^A \left(-\frac{1}{2} X_e^I X_f^I + X_e^4 X_f^4 + X_e^8 X_f^8 \right) + X_a^8 X_b^4 X_e^I X_f^I \right] \\
& - \frac{g_M^4 v^2 N}{128} \left[(X_a^I X_b^I)^2 + (X_a^A X_b^{A+4} - X_a^{A+4} X_b^A)^2 - \frac{5}{8} (X_a^I X_a^I)^2 \right. \\
& + X_a^I X_a^I ((X_b^4)^2 + (X_b^8)^2) - 2(X_a^{A+4} X_b^4 + X_a^A X_b^8) (X_b^{A+4} X_a^4 + X_b^A X_a^8) \\
& \left. - 2(X_a^A X_b^4 - X_a^{A+4} X_b^8) (X_b^A X_a^4 - X_b^{A+4} X_a^8) \right]. \quad (B.3)
\end{aligned}$$

If we further restrict to pieces not involving the ‘special’ directions 4 and 8, which we will denote with indices I'' , the $\mathcal{O}(v^2)$ potential looks very simple:

$$V_{v^2}^{\text{SU}(N), I''} = -\frac{g_M^4 v^2 N}{128} [X_a^{I''} X_b^{I''} X_a^{J''} X_b^{J''} + (X_a^{A'} X_b^{A'+4} - X_a^{A'+4} X_b^{A'})^2 - \frac{5}{8} (X_a^{I''} X_a^{I''})^2] \\ - \frac{g_M^2 v^2 N \lambda}{16} f^{abc} d^{ef}{}_c X_a^{A'+4} X_b^{A'} X_e^{I''} X_f^{I''}, \quad (\text{B.4})$$

where $A' = 1, 2, 3$.

B.2 Fermionic terms

The full fermionic terms, including the ones of the same order as the above bosonic terms, are

$$\begin{aligned} & \frac{i}{2} f_{AB}^\mu \bar{\chi}_\mu^{AB} (v^2 N + v N \sqrt{2} X_0^4 + \frac{1}{2} (N X_0^I X_0^I + X_a^I X_a^I)) \\ & + \frac{iv}{\sqrt{2}} \lambda \epsilon^{\mu\nu\rho} (\bar{\chi}_{\mu AB} \gamma_\nu \chi_\rho^{AB}) f^{abc} X_a^{C'+4} X_b^{C'} X_c^8 \\ & + \frac{i}{64} g_M^2 \epsilon^{\mu\nu\rho} (\bar{\chi}_{\nu AB} \gamma_\rho \chi_\mu^{AB}) (v^2 N + v N \sqrt{2} X_0^4) \left[v^2 N + v N \sqrt{2} X_0^4 + \frac{1}{2} (N X_0^I X_0^I + X_a^I X_a^I) \right] \\ & + \frac{i}{16} g_M^2 e \left[N (\bar{\psi}_0^D - i \bar{\psi}_0^{D+4}) (\psi_0^D + i \psi_0^{D+4}) \right. \\ & \quad \left. + (i \bar{\psi}_a^D + \bar{\psi}_a^{D+4}) (-i \psi_a^D + \psi_a^{D+4}) \right] (v^2 N + v N \sqrt{2} X_0^4) \\ & + i \epsilon^{\mu\nu\rho} \bar{\chi}_\mu^{AC} \chi_{\nu BC} \left[\frac{v N}{\sqrt{2}} \delta^{B,4} \partial_\rho (X_0^A - i X_0^{A+4}) + \sqrt{2} v (X_a^B + i X_a^{B+4}) A_\rho^{a-} \delta^{A,4} \right. \\ & \quad \left. + \frac{N}{2} (X_0^B + i X_0^{B+4}) \partial_\rho (X_0^A - i X_0^{A+4}) + \frac{(X_a^{B+4} - i X_a^B) D_\rho (X_a^{A+4} + i X_a^A)}{2} \right. \\ & \quad \left. - i N \delta^{A,4} A_{\mu 0}^- [2 v^2 \delta_{B,4} + \sqrt{2} (X_0^B + i X_0^{B+4})] \right] + c.c. \\ & - \frac{i}{4} g_M^2 e \left[N (\bar{\psi}_0^D - i \bar{\psi}_0^{D+4}) (\psi_0^B + i \psi_0^{B+4}) + (i \bar{\psi}_a^D + \bar{\psi}_a^{D+4}) (-i \psi_a^B + \psi_a^{B+4}) \right] \times \\ & \times \left[v^2 N \delta_{B,4} \delta^{D,4} + \frac{v N}{\sqrt{2}} ((X_0^B - i X_0^{B+4}) \delta^{D,4} + \delta_{B,4} (X_0^D + i X_0^{D+4})) \right] \\ & - \frac{i}{4} g_M^2 \epsilon^{\mu\nu\rho} (\bar{\chi}_{\nu AB} \gamma_\rho \chi_\mu^{CD}) \left[v^2 N \delta_{C,4} \delta^{A,4} + \frac{v N}{\sqrt{2}} ((X_0^C - i X_0^{C+4}) \delta^{A,4} + \delta_{C,4} (X_0^A + i X_0^{A+4})) \right] \times \\ & \times \left[v^2 N \delta_{D,4} \delta^{B,4} + \frac{v N}{\sqrt{2}} ((X_0^D - i X_0^{D+4}) \delta^{B,4} + \delta_{D,4} (X_0^B + i X_0^{B+4})) \right] \\ & + i A e \bar{\chi}_\mu^{BA} \gamma^\mu \gamma^\nu \left\{ \frac{N}{\sqrt{2}} (\psi_0^{[A} + i \psi_0^{[A+4}) D_\mu (X_0^{B]} - i X_0^{B]+4}) \right. \\ & \quad \left. + (i \psi_a^{[A} - \psi_a^{[A+4}) \left[\frac{D_\mu (-i X_a^{B]} - X_a^{B]+4})}{\sqrt{2}} - 2 i v A_{\mu a}^- \delta^{B,4} \right] \right\} + c.c. \\ & + \frac{A^2}{2} e \bar{\chi}_\mu^{BA} \gamma^\nu \gamma^\mu (\bar{\chi}_{\nu BC})^\dagger \left[N (\psi_0^A + i \psi_0^{A+4}) (\psi_0^C - i \psi_0^{C+4}) \right. \\ & \quad \left. + (i \psi_a^A - \psi_a^{A+4}) (-i \psi_a^C - \psi_a^{C+4}) \right] + c.c. \\ & - i A \bar{f}^{\mu AB} \gamma_\mu \left[v N \delta_{B,4} (\psi_0^A + i \psi_0^{A+4}) + \frac{N}{\sqrt{2}} (\psi_0^A + i \psi_0^{A+4}) (X_0^B - i X_0^{B+4}) \right. \\ & \quad \left. + \frac{1}{\sqrt{2}} (i \psi_a^A - \psi_a^{A+4}) (-i X_a^B - X_a^{B+4}) \right] + c.c. \end{aligned}$$

$$\begin{aligned}
& -\frac{i}{16}g_M^2 e A \bar{\chi}_{\mu AB} \gamma^\mu \left[v N \delta^{4A} (\psi_0^B - i\psi_0^{B+4}) + \frac{N}{\sqrt{2}} (\psi_0^B - i\psi_0^{B+4}) (X_0^A + iX_0^{A+4}) \right. \\
& + \frac{1}{\sqrt{2}} (-i\psi_a^B - \psi_a^{B+4}) (iX_a^A - X_a^{A+4}) \left. \right] (v^2 N + v N \sqrt{2} X_0^4) + c.c. \\
& -\frac{ie}{4}g_M^2 A \bar{\chi}_{\mu AB} \gamma^\mu \left[v N \delta^{B4} (\psi_0^D - i\psi_0^{D+4}) + \frac{N}{\sqrt{2}} (\psi_0^D - i\psi_0^{D+4}) (X_0^B + iX_0^{B+4}) \right. \\
& + \frac{1}{\sqrt{2}} (-i\psi_a^D - \psi_a^{D+4}) (iX_a^B - X_a^{B+4}) \left. \right] \left[v^2 N \delta_{D,4} \delta^{A,4} + \frac{vN}{\sqrt{2}} ((X_0^D - iX_0^{D+4}) \delta^{A,4} \right. \\
& \quad \left. + \delta_{D,4} (X_0^A + iX_0^{A+4})) \right] + c.c. \\
& -\frac{i}{4}g_M^2 \epsilon^{\mu\nu\rho} (\bar{\chi}_{\nu AB} \gamma_\rho \chi_\mu^{CD}) \left[v^2 N \delta_{C,4} \delta^{A,4} + \frac{vN}{\sqrt{2}} ((X_0^C - iX_0^{C+4}) \delta^{A,4} \right. \\
& \quad \left. + \delta_{C,4} (X_0^A + iX_0^{A+4})) \right] \times \\
& \times \left[v^2 N \delta_{D,4} \delta^{B,4} + \frac{vN}{\sqrt{2}} ((X_0^D - iX_0^{D+4}) \delta^{B,4} + \delta_{D,4} (X_0^B + iX_0^{B+4})) \right] \\
& + \lambda \frac{v}{\sqrt{2}} \epsilon^{\mu\nu\rho} (\bar{\chi}_{\mu AB} \gamma_\nu \chi_\rho^{CD}) f^{abc} \left[(iX_a^{[C} + X_a^{[C+4}) (iX_b^{D]} + X_b^{D]+4}) (iX_c^{[A} - X_c^{[A+4}) \delta^{B],4} \right. \\
& + (iX_a^{[A} - X_a^{[A+4}) (iX_b^{B]} - X_b^{B]+4}) (iX_c^{[C} + X_c^{[C+4}) \delta^{D],4} \left. \right] \\
& + \lambda e A \bar{\chi}_{\mu AB} \gamma^\mu f^{abc} \left[v (i\psi_c^B - \psi_c^{B+4}) X_a^8 (iX_b^A - X_b^{A+4}) - i v \delta^{A,4} X_a^{D+4} X_b^D (i\psi_c^A - \psi_c^{A+4}) \right] \\
& - \lambda e A \bar{\chi}_{\mu AB} \gamma^\mu f^{abc} \left[\frac{1}{2} v (i\psi_c^4 - \psi_c^8) (iX_a^{[A} - X_a^{[A+4}) (iX_b^{B]} - X_b^{B]+4}) \right. \\
& + v (i\psi_c^D - \psi_c^{D+4}) \delta^{[A,4} (-iX_{a,D} - X_{a,D+4}) (iX_b^{B]} - X_b^{B]+4}) \left. \right] + \text{subleading} . \tag{B.5}
\end{aligned}$$

Restricting to the terms of the same order as the bosonic $\mathcal{O}(v^2)$ terms, and also to the $SU(N)$ component, we obtain

$$\begin{aligned}
& \frac{i}{4} f_{AB}^\mu \chi_\mu^{AB} X_a^I X_a^I + \frac{iv}{\sqrt{2}} \lambda \epsilon^{\mu\nu\rho} (\bar{\chi}_{\mu AB} \gamma_\nu \chi_\rho^{AB}) f^{abc} X_a^{C'+4} X_b^{C'} X_c^8 \\
& + i \epsilon^{\mu\nu\rho} \bar{\chi}_\mu^{AC} \chi_{\nu BC} \left[\sqrt{2} v (X_a^B + iX_a^{B+4}) A_\rho^a - \delta^{A,4} + \frac{1}{2} (X_a^{B+4} - iX_a^B) D_\rho (X_a^{A+4} + iX_a^A) \right] + c.c. \\
& + i A e \bar{\chi}_\mu^{BA} \gamma^\mu \gamma^\nu (i\psi_a^{[A} - \psi_a^{[A+4}) \left[\frac{1}{\sqrt{2}} D_\mu (-iX_a^{B]} - X_a^{B]+4}) - 2 i v A_{\mu a}^- \delta^{B,4} \right] + c.c. \\
& + \frac{A^2}{2} e \bar{\chi}_\mu^{BA} \gamma^\nu \gamma^\mu (\bar{\chi}_{\nu BC})^\dagger (i\psi_a^A - \psi_a^{A+4}) (-i\psi_a^C - \psi_a^{C+4}) + c.c. \\
& - i A \bar{f}^{\mu AB} \gamma_\mu \frac{1}{\sqrt{2}} (i\psi_a^A - \psi_a^{A+4}) (-iX_a^B - X_a^{B+4}) + c.c. \\
& + \lambda \frac{v}{\sqrt{2}} \epsilon^{\mu\nu\rho} (\bar{\chi}_{\mu AB} \gamma_\nu \chi_\rho^{CD}) f^{abc} \left[(iX_a^{[C} + X_a^{[C+4}) (iX_b^{D]} + X_b^{D]+4}) (iX_c^{[A} - X_c^{[A+4}) \delta^{B],4} \right. \\
& + (iX_a^{[A} - X_a^{[A+4}) (iX_b^{B]} - X_b^{B]+4}) (iX_c^{[C} + X_c^{[C+4}) \delta^{D],4} \left. \right] \\
& + \lambda v e A \bar{\chi}_{\mu AB} \gamma^\mu f^{abc} \left[(i\psi_c^B - \psi_c^{B+4}) X_a^8 (iX_b^A - X_b^{A+4}) - i \delta^{A,4} X_a^{D+4} X_b^D (i\psi_c^A - \psi_c^{A+4}) \right] \\
& - \lambda v e A \bar{\chi}_{\mu AB} \gamma^\mu f^{abc} \left[\frac{1}{2} (i\psi_c^4 - \psi_c^8) (iX_a^{[A} - X_a^{[A+4}) (iX_b^{B]} - X_b^{B]+4}) \right. \\
& + (i\psi_c^D - \psi_c^{D+4}) \delta^{[A,4} (-iX_{a,D} - X_{a,D+4}) (iX_b^{B]} - X_b^{B]+4}) \left. \right] + \text{subleading} . \tag{B.6}
\end{aligned}$$

If we further restrict only to the I'' indices, that is we again exclude contributions from the directions 4 and 8, we get

$$\frac{i}{4} f_{A'B'}^\mu \chi_\mu^{A'B'} X_a^{I''} X_a^{I''} + i \epsilon^{\mu\nu\rho} \bar{\chi}_\mu^{A'C'} \chi_{\nu B'C'} \frac{1}{2} (X_a^{B'+4} - iX_a^{B'}) D_\rho (X_a^{A'+4} + iX_a^{A'}) + c.c.$$

$$\begin{aligned}
& + iAe\bar{\chi}_\mu^{B'A'}\gamma^\mu\gamma^\nu(i\psi_a^{[A'} - \psi_a^{A'+4]}\Big[\frac{1}{\sqrt{2}}D_\mu(-iX_a^{B']} - X_a^{B'+4})\Big] + c.c. \\
& + \frac{A^2}{2}e\bar{\chi}_\mu^{B'A'}\gamma^\nu\gamma^\mu(\bar{\chi}_{\nu B'C'})^\dagger(i\psi_a^{A'} - \psi_a^{A'+4})(-i\psi_a^{C'} - \psi_a^{C'+4}) + c.c. \\
& - iA\bar{f}^{\mu A'B'}\gamma_\mu\frac{1}{\sqrt{2}}(i\psi_a^{A'} - \psi_a^{A'+4})(-iX_a^{B'} - X_a^{B'+4}) + c.c. + \text{subleading} .
\end{aligned} \tag{B.7}$$

References

- [1] O. Aharony, O. Bergman, D. L. Jafferis, and J. Maldacena, *N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, *JHEP* **10** (2008) 091, [[arXiv:0806.1218](#)].
- [2] S. Mukhi and C. Papageorgakis, *M2 to D2*, *JHEP* **05** (2008) 085, [[arXiv:0803.3218](#)].
- [3] J. Bagger and N. Lambert, *Modeling multiple M2's*, *Phys. Rev.* **D75** (2007) 045020, [[hep-th/0611108](#)].
- [4] J. Bagger and N. Lambert, *Gauge Symmetry and Supersymmetry of Multiple M2-Branes*, *Phys. Rev.* **D77** (2008) 065008, [[arXiv:0711.0955](#)].
- [5] J. Bagger and N. Lambert, *Comments On Multiple M2-branes*, *JHEP* **02** (2008) 105, [[arXiv:0712.3738](#)].
- [6] A. Gustavsson, *Algebraic structures on parallel M2-branes*, *Nucl. Phys.* **B811** (2009) 66–76, [[arXiv:0709.1260](#)].
- [7] T. Li, Y. Liu, and D. Xie, *Multiple D2-Brane Action from M2-Branes*, [arXiv:0807.1183](#).
- [8] Y. Pang and T. Wang, *From N M2's to N D2's*, *Phys. Rev.* **D78** (2008) 125007, [[arXiv:0807.1444](#)].
- [9] X. Chu and B. E. W. Nilsson, *Three-dimensional topologically gauged N=6 ABJM type theories*, [arXiv:0906.1655](#).
- [10] U. Gran and B. E. W. Nilsson, *Three-dimensional N=8 superconformal gravity and its coupling to BLG M2-branes*, *JHEP* **03** (2009) 074, [[arXiv:0809.4478](#)].
- [11] W. Li, W. Song, and A. Strominger, *Chiral Gravity in Three Dimensions*, *JHEP* **04** (2008) 082, [[arXiv:0801.4566](#)].
- [12] S. Deser, R. Jackiw, and S. Templeton, *Topologically massive gauge theories*, *Ann. Phys.* **140** (1982) 372–411.
- [13] E. A. Bergshoeff, O. Hohm, J. Rosseel, E. Sezgin, and P. K. Townsend, *More on Massive 3D Supergravity*, [arXiv:1005.3952](#).
- [14] A. Maloney, W. Song, and A. Strominger, *Chiral Gravity, Log Gravity and Extremal CFT*, *Phys. Rev.* **D81** (2010) 064007, [[arXiv:0903.4573](#)].
- [15] K. Skenderis, M. Taylor, and B. C. van Rees, *Topologically Massive Gravity and the AdS/CFT Correspondence*, *JHEP* **09** (2009) 045, [[arXiv:0906.4926](#)].
- [16] S. Deser and J. Franklin, *Is BTZ a separate superselection sector of CTMG?*, [arXiv:1007.2637](#).
- [17] B. E. Nilsson, *Light-cone analysis of ungauged and topologically gauged BLG theories*, *Class.Quant.Grav.* **26** (2009) 175001, [[arXiv:0811.3388](#)].

- [18] H. Liu and A. A. Tseytlin, *D = 4 super Yang-Mills, D = 5 gauged supergravity, and D = 4 conformal supergravity*, *Nucl. Phys.* **B533** (1998) 88–108, [[hep-th/9804083](#)].
- [19] G. R. Dvali, G. Gabadadze, and M. Porrati, *4D gravity on a brane in 5D Minkowski space*, *Phys. Lett.* **B485** (2000) 208–214, [[hep-th/0005016](#)].
- [20] M. Becker, P. Bruillard, and S. Downes, *Chiral Supergravity*, *JHEP* **10** (2009) 004, [[arXiv:0906.4822](#)].
- [21] D. Grumiller and N. Johansson, *Gravity duals for logarithmic conformal field theories*, [arXiv:1001.0002](#).
- [22] D. Grumiller, N. Johansson, and T. Zojer, *Short-cut to new anomalies in gravity duals to logarithmic conformal field theories*, [arXiv:1010.4449](#).
- [23] E. Bergshoeff, E. Sezgin, and P. K. Townsend, *Supermembranes and eleven-dimensional supergravity*, *Phys. Lett.* **B189** (1987) 75–78.
- [24] N. Lambert and P. Richmond, *M2-Branes and Background Fields*, *JHEP* **10** (2009) 084, [[arXiv:0908.2896](#)].
- [25] Y. Kim, O.-K. Kwon, H. Nakajima, and D. D. Tolla, *Interaction between M2-branes and Bulk Form Fields*, [arXiv:1009.5209](#).
- [26] M. Schnabl and Y. Tachikawa, *Classification of N=6 superconformal theories of ABJM type*, [arXiv:0807.1102](#).
- [27] J. Palmkvist, *Three-algebras, triple systems and 3-graded Lie superalgebras*, *J. Phys.* **A43** (2010) 015205, [[arXiv:0905.2468](#)].
- [28] N. Lambert and C. Papageorgakis, *Relating $U(N)xU(N)$ to $SU(N)xSU(N)$ Chern-Simons Membrane theories*, [arXiv:1001.4779](#).
- [29] A. Gustavsson, *Monopoles, three-algebras and ABJM theories with $N = 5, 6, 8$ supersymmetry*, [arXiv:1012.4568](#).
- [30] J. Bagger and N. Lambert, *Three-Algebras and N=6 Chern-Simons Gauge Theories*, *Phys. Rev.* **D79** (2009) 025002, [[arXiv:0807.0163](#)].
- [31] B. E. W. Nilsson and J. Palmkvist, *Superconformal M2-branes and generalized Jordan triple systems*, *Class. Quant. Grav.* **26** (2009) 075007, [[arXiv:0807.5134](#)].
- [32] M. Van Raamsdonk, *Comments on the Bagger-Lambert theory and multiple M2- branes*, *JHEP* **05** (2008) 105, [[arXiv:0803.3803](#)].
- [33] J. Distler, S. Mukhi, C. Papageorgakis, and M. Van Raamsdonk, *M2-branes on M-folds*, *JHEP* **05** (2008) 038, [[arXiv:0804.1256](#)].
- [34] S. Terashima, *On M5-branes in N=6 Membrane Action*, *JHEP* **08** (2008) 080, [[arXiv:0807.0197](#)].
- [35] P. K. Townsend, K. Pilch, and P. van Nieuwenhuizen, *Selfduality in Odd Dimensions*, *Phys. Lett.* **136B** (1984) 38.
- [36] S. Deser and R. Jackiw, *'Selfduality' of Topologically Massive Gauge Theories*, *Phys. Lett.* **B139** (1984) 371.
- [37] P. K. Townsend, K. Pilch, and P. van Nieuwenhuizen, *Selfduality in Odd Dimensions: Addendum*, *Phys. Lett.* **137B** (1984) 443.

- [38] H. Nastase, D. Vaman, and P. van Nieuwenhuizen, *Consistent nonlinear $K K$ reduction of 11d supergravity on $AdS(7) \times S(4)$ and self-duality in odd dimensions*, *Phys. Lett.* **B469** (1999) 96–102, [[hep-th/9905075](#)].
- [39] H. Nastase, D. Vaman, and P. van Nieuwenhuizen, *Consistency of the $AdS(7) \times S(4)$ reduction and the origin of self-duality in odd dimensions*, *Nucl. Phys.* **B581** (2000) 179–239, [[hep-th/9911238](#)].
- [40] M. A. Bandres, A. E. Lipstein, and J. H. Schwarz, *Studies of the ABJM Theory in a Formulation with Manifest $SU(4)$ R-Symmetry*, [arXiv:0807.0880](#).
- [41] M. J. Duff, B. E. W. Nilsson, and C. N. Pope, *Kaluza-Klein Supergravity*, *Phys. Rept.* **130** (1986) 1–142.
- [42] H. J. Kim, L. J. Romans, and P. van Nieuwenhuizen, *The Mass Spectrum of Chiral $N=2$ $D=10$ Supergravity on S^{*5}* , *Phys. Rev.* **D32** (1985) 389.
- [43] K. Skenderis, M. Taylor, and B. C. van Rees, *AdS boundary conditions and the Topologically Massive Gravity/CFT correspondence*, [arXiv:0909.5617](#).
- [44] J. H. Horne and E. Witten, *Conformal Gravity in three-dimensions as a Gauge Theory*, *Phys. Rev. Lett.* **62** (1989) 501–504.
- [45] W. D. Goldberger and M. B. Wise, *Modulus stabilization with bulk fields*, *Phys. Rev. Lett.* **83** (1999) 4922–4925, [[hep-ph/9907447](#)].
- [46] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690–4693, [[hep-th/9906064](#)].
- [47] J. A. Harvey and A. B. Royston, *Localized Modes at a D -brane– O -plane Intersection and Heterotic Alice Strings*, *JHEP* **04** (2008) 018, [[arXiv:0709.1482](#)].
- [48] J. A. Harvey and A. B. Royston, *Gauge/Gravity duality with a chiral $N=(0,8)$ string defect*, *JHEP* **08** (2008) 006, [[arXiv:0804.2854](#)].